Full paper

UTILIZATION OF WEIBULL TECHNIQUES TO DESCRIBE STABILITY DISTRIBUTION OF CARBON RESIN ELECTRODES

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ABSTRACT

As a follow up to previous studies, in this paper, the stability of carbon-resin electrodes has been statistically analysed using Weibull distribution. Carbon electrodes were developed from used and discarded dry cells and resin using a non-heat treatment process. Stability of the electrodes during and after electrochemical treatment was tested in synthetic wastewater prepared using standard methods. Effects of particle size, percentage of resin and compaction pressure on the stability of the electrodes were studied and a mathematical model was developed. Results of the stability tests were analysed using Weibull techniques. The techniques were evaluated using known statistical methods (total error, coefficient of determination and model selection criterion) .

The study revealed that the electrodes had a mean of stability after 24 hours of 89.79 % with standard deviation and coefficient of variation of 2.57 and 2.86 % respectively. The mathematical model of stability in use with respect to particle size (S_a), percentage binder (P_c) and compaction pressure (C_u) revealed that stability of the electrodes depends mainly on S_a (2.38 x 10^{4} /m) and C_u (5.82x 10^{-7} m^2/N), but less on P_c (3.84/%). Correlation coefficient between experimental and predicted stabilities using the model was found to be statistically adequate indicating that there is an agreement between the values statistically. Weibull distribution parameters namely scale ' b' and shape 'c' parameters were in the range of 88.99 to 92.49 and 31.80 to 50.62 respectively. This indicates that selected operational factors affect stability distribution of carbon resin electrodes and that at compaction pressure of greater than 90 MN/m^2 the electrodes will be stable. Statistical evaluation showed that total error, coefficient of determination and model selection criterion were 8.321, 0.998 and 5.739, and 0.606, 1.000 and 11.496 for three and two parameter models respectively. These results show that two parameter model is better than three parameters model because of lower total error, higher coefficient of determination and model of selection criterion. An application of limit state criterion provides information about safe and unsafe regions for the stability, particle size and compaction pressure.

It was concluded that Weibull distribution can be applied in development of carbon resin electrodes, which are important ingredients in wastewaters treatment. Mathematical model developed allows researchers to describe the stability of a carbon resin electrodes and provides information for the manufacturers that will help in the production of the electrodes.

KEYWORDS: Weibull distribution, Stability, Wastewaters, Carbon-resin electrodes, Mathematical modelling

I. INTRODUCTION

Electrodes are not isotropic materials (Dirikolu et al, 2002). They have particular stability properties which vary due to their internal structure. This variation in stability leads to the necessity of employing statistical and other techniques in the design, development and production of the electrodes. One of these techniques is the Weibull distribution, which has recently been used for the determination of static and dynamic properties of ceramics and metal-matrix, ceramic-matrix, and polymer-matrix composites (Dirikolu et al, 2002). Weibull distribution has the capability to model experimental data of very different character. This is one of the reasons for its wide utilisation nowadays. In recent years, research papers and books dealing with the historical development and application of this statistical method have been published (Dirikolu et al, 2002). Dodson (1994) described the developments regarding the estimation approaches for Weibull distribution parameters and there has been considerable work on new application areas and improved estimation approaches. Barbero et al. (2000) applied this technique in the modelling of mechanical properties of composite materials and suggested the Weibull distribution as a practical method in the determination of 90% and 95% reliability and stability of composite materials (Dirikolu et al, 2002). Oke (2008) successfully applied the Weibull distribution in the design of environmental pollution control devices.

Stability can be described as resistance of any substance to retain its initial properties under a specified environmental condition. In other words stability can be explained as a measure of resistance to total collapse or failure of the substance. It can be measured in terms of weight, diameter or length of the substance and it can be measured as a function of a particular process in specific solutions (Oke et al, 2007a and b; 2009a and b). In previous works by some of the authors, carbon- resin electrodes were developed and their stabilities were also determined for use in electrochemical treatment processes of wastewaters (Oke et al, 2007a and b) while the importance of carbon resin electrodes during electrochemical treatment process was listed. Considering the importance of electrochemical treatment process in environmental pollution control stated in literature such as Chen (2004); Oke (2007a) and the importance of carbon electrodes during electrochemical treatment process and cost of producing carbon electrodes using heat treatment processes, there is a need to describe stability of carbon- resin electrodes using Weibull distribution. The main objective of this current study is to describe stability of carbon resin electrodes with a particular attention to stability distribution using Weibull techniques and to evaluate each of the models statistically

2. MATERIALS AND METHOD

Discarded dry cells (size D R20 UM-1) were collected from different towns in Oyo and Osun States, Nigeria. These collected dry cells were sectioned; and graphite (carbon) inside them were removed and pulverised. The powdered graphite was sieved into different particle sizes. A known mass of the powdered graphite was mixed with an organic binder, moulded into 2.5 cm diameter and 10 cm long electrode using locally fabricated equipment. The graphite rod so developed was used as anode while an aluminum rod was used as a cathode for electrochemical treatment of synthetic wastewater (prepared by dissolving 24,000 mg of analytical Sodium Chloride in 1000 ml of distilled water; APHA, 1998) to determine the stability of the electrode during and after the electrochemical process (Figure 1). Effects of compaction pressure, particle size and percentage of binder on the stability of the electrode were studied. Selection of these operational factors was based on previous studies (Oke et al, 2007a, b and c; 2009a) and lack of data that describe stability characteristics of carbon-resin electrodes. Computation of the stability of the electrodes based on retained weight, was determined using Equation (1).

$$x(\%) = \frac{M_{blamk} - \Delta M_{sample}}{M_{initial}} \text{ x } 100 \qquad (1)$$

Results of the stability tests conducted were analyzed using Weibull techniques and a mathematical model of the stability in relation to the three selected operational factors used was developed. Statistical evaluation of the Weibull techniques and mathematical model was conducted using total error, coefficient of determination (CD) and model selection criterion (MSC).

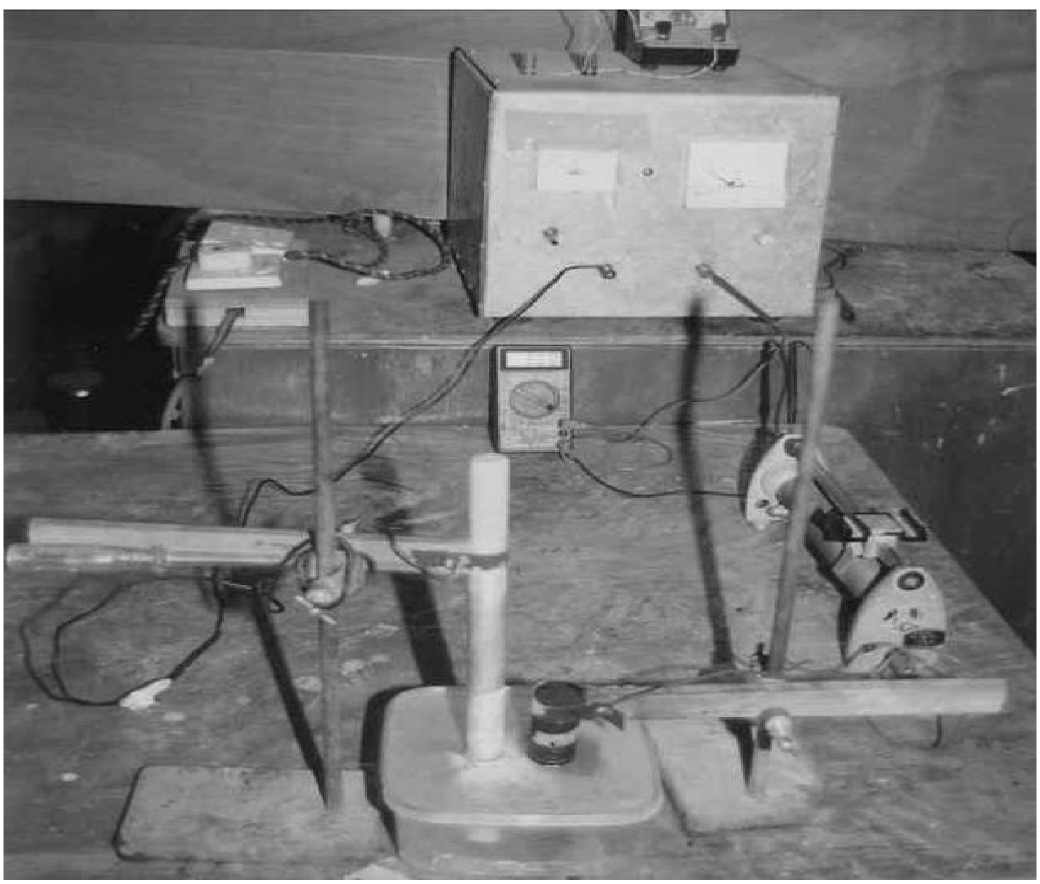

Figure 1 Laboratory set-up for the studies.

3. RESULTS AND DISCUSSION

Results from the study will be presented in three categories as follows:

i. Stability of carbon resin electrodes during and after electrochemical treatment
ii. Distributions of the stability
iii. Statistical evaluation of the distribution models

3.1. Stability of carbon resin electrodes during and after electrochemical treatment

The results obtained from the experiments in the present work are presented in figures 2 to 7. Figure 2 presents loss in weight per initial weight of the electrodes, while Figure 3 shows stability during electrochemical process with respect to time. From Fig.3 it is observed that stability of the electrodes decreases with increasing time. This phenomenon can be attributed to loss in weight of the electrodes due to reduction in cohesive forces between carbon particles and the binder (Fig.2). Figures 4-7 follow the same trend of increasing loss of weight or decreasing stability of the electrodes with respect to time. From the figures (Fig. 4 to 7) it can be seen that there are relationships between stability of the electrodes, loss in weight of the electrodes and time at various selected operational factors. Figures 4 and 5 present relationship between loss in weight, stability of the electrodes and time at different percentages of the binder. Figures 6 and 7 show relationships between loss in weight, stability of the electrodes and time at different particle size of carbon. From the figures it is observed that loss in weight per unit initial weight of the electrodes increases with increasing time, decreasing percentage of the binder and with increasing particle size. On the other hand, stability of the electrodes increases with increasing percentage of the binder, decreasing time and decreasing particle size. These phenomena can be attributed to decrease in contact surface area (with increasing particle size), loss in weight per unit weight and reduction in cohesive forces. Table 1 presents stability of the electrodes after electrochemical process of 24 hours and at selected levels of the operational factors. From the table it can be seen that the stability of the electrodes after 24 hours run ranges from 87.8 to 93.4 % with a mean and standard deviation of 89.79% and 2.57 respectively with coefficient of variation being 2.86 %. These results indicate that stability of the electrodes is high and has little variation at lower carbon particle size, higher percentage of the binder and compaction pressure.

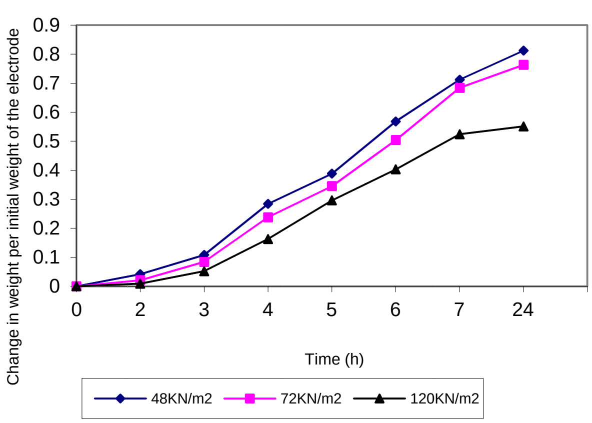

Figure 2 Variation of loss in weight of the electrode with time for 0.118 mm particle size and 0.5 % binder at various compaction pressures

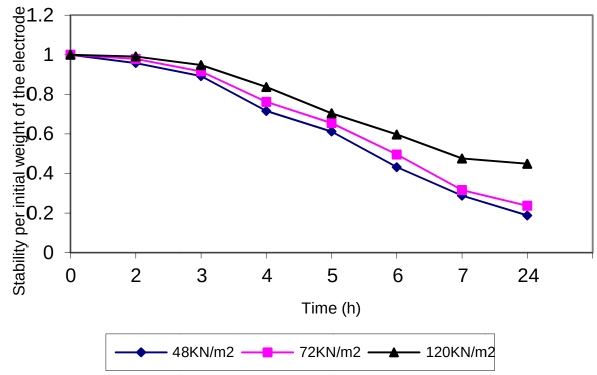

Figure 3 Variation of stability of the electrode with time for 0.118 mm particle size and 0.5 % binder at various compaction pressures

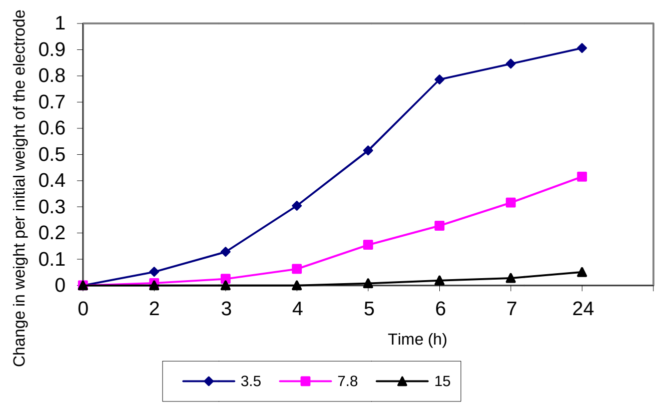

Figure 4. Variation of loss in weight of the electrode with time for 0.254 mm particle size and 120 kN/m^2 compaction pressure at various percentage of binder

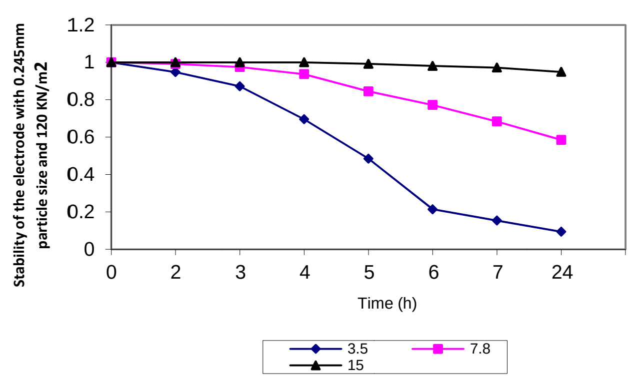

Figure 5. Variation of stability of the electrode with time for 0.254 mm particle size and 120 kN/m^2 compaction pressure at various percentage of binder

In order to establish statistical relationship between these selected operational factors and stability of the electrodes a mathematical model was developed through the use of least squares method. Mathematical model of the relationship between stability and the three operational factors is Y (stability in use) = 3.84 P_c +0.582 C_u – 0.0238S_a. This means that the mathematical model of stability in use with respect to particle size (S_a), percentage binder (P_c) and compaction pressure (C_u) revealed that stability of the

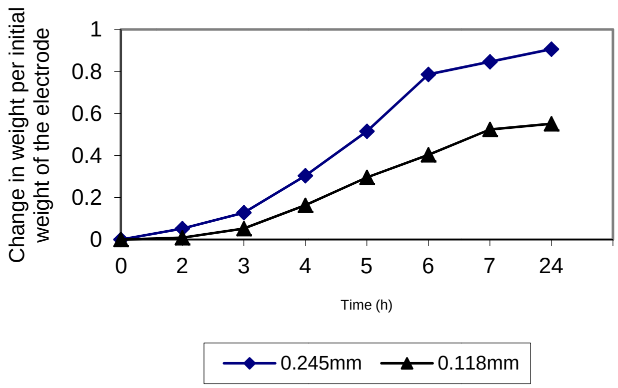

Figure 6. Variation of loss in weight of the electrode with time for 3.5 % binder and 120 kNm^{-2} compaction pressure at various particle sizes

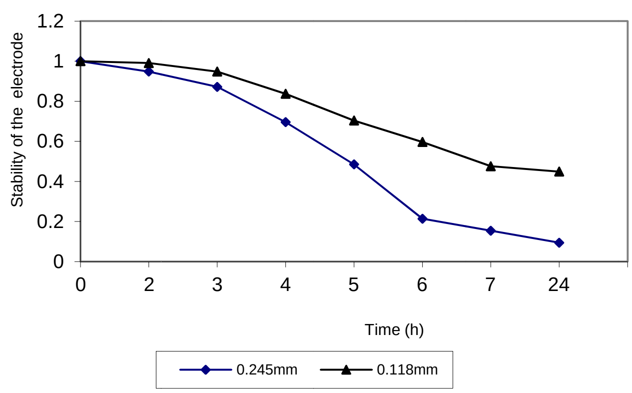

Figure 7. Variation of stability of the electrode with time for 3.5 % binder and 120 kNm^{-2} compaction pressure at various particle sizes

electrodes depends mainly on S_a (2.38 x 10 ^{4} /m) and C_u (5.82x 10^{-7}m^2/N), but less on P_c (3.84/%). This result indicates that stability in use depends highly on percentage of binder, followed by compaction pressure and with particle size as the least. Influence of particle size was negative (reduces stability with higher particle size) while influences of compaction pressure and percentage binder used were positive, indicating that the higher these factors the higher the stability. Statistical evaluation using correlation coefficient (R^2) indicates that there is a reasonable agreement between experimental stabilities and predicted stabilities using the mathematical model (Fig. 8)

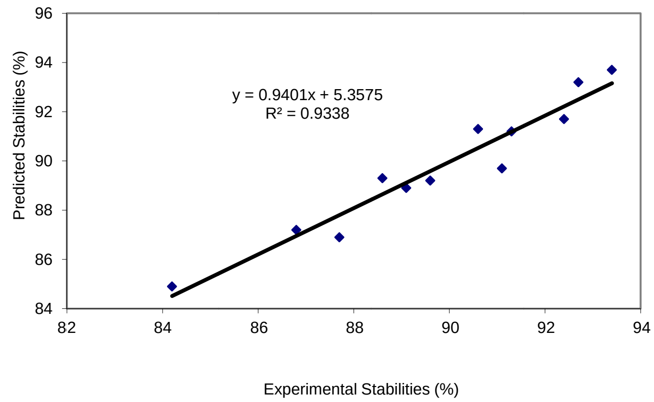

Figure 8. Statistical evaluation of experimental stabilities and predicted stabilities

3.2. Distributions of the stability

Weibull distribution is being used to model extreme values such as failure times and stability. Two popular forms of this distribution are two- and three-parameter Weibull distributions. The distribution function of the three parameter Weibull distribution is given as follows (Chow et al, 1988; Dirikolu et al, 2002):

$$f(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (2)$$

where; *a*, *b*, and *c* are location, scale and shape parameters respectively (Dirikolu et al, 2002). When *a* = 0 in Eq. (2) the distribution function of the two-parameter Weibull distribution is obtained. The three-parameter Weibull distribution is suitable for

Table 1 Stability of the electrodes in synthetic wastewater after 24 hours of electrochemical process

Particle size (μm)	Percentage of the binder (%)	Compaction pressure (MN/m^2)	Experimental Stability	Predicted Stability using mathematical model of Y (stability in use) = 3.84 P_c +0.582 C_u – 0.0238S_a
45	8	110	93.4	93.7
63	8	110	92.7	93.2
150	8	110	91.3	91.2
245	8	110	89.1	88.9
45	10	90	91.1	89.7
63	10	90	88.6	89.3
150	10	90	86.8	87.2
245	10	90	84.2	84.9
45	9	100	92.4	91.7
63	9	100	90.6	91.3
150	9	100	89.6	89.2
245	9	100	87.7	86.9

situations in which an extreme value cannot take values less than *a*. In this study, the three and two-parameter Weibull distribution were used. The distribution function in two parameter case can be written as follows:

$$F(x) = 1-\exp\left[-\left(\frac{x}{b}\right)^{c}\right] \quad (3)$$

In the context of this study, f(*x*; *b*; *c*), represents the probability that the stability is equal to or less than *x*. Using the equality f (*x*; *b*; *c*) + *R*(*x*; *b*; *c*) = 1 , the reliability *R*(*x*; *b*, *c*), that is, the probability that the stability is at least *x*, is defined as (Chow et al, 1988; Dirikolu et al, 2002)

$$R(x) = \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \quad (4)$$

$$R(x) = \exp\left[-\left(\frac{x}{b}\right)^{c}\right] \quad (5)$$

The parameters *b* and *c* of the distribution function f(*x*; *b*; *c*) are estimated from observations. The methods usually employed in the estimation of these parameters are method of linear regression (least squares method), method of maximum likelihood, and method of moments (Dirikolu et al, 2002). Among these methods, the use of linear regression goes back to the days when computers were not available: the linear regression line was plotted manually with the help of Weibull graph papers. Linear regression is still common among practitioners, and will be used for parameter estimation in this paper. The linear regression method is based on transforming Eq. (2) or eq (3) into equations (6 and 7) and taking the double logarithms of both sides. Hence, a linear regression Y = mx + C as shown in equations (8 and 9) can be derived for three and two parameters respectively.

$$\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] = 1-F(x) \quad (6)$$

$$\exp\left[-\left(\frac{x}{b}\right)^{c}\right] = 1-F(x) \quad (7)$$

$$Log_e\left(Log_e\left(1-F(x)\right)^{-1}\right)= cLog_e\left(x-a\right)-cLog_e b \quad (8)$$

$$Log_e\left(Log_e\left(1-F(x)\right)^{-1}\right)= cLog_e x-cLog_e b \quad (9)$$

It is well known that f(x; b; c) is an unknown in (4a and b) and can be estimated from observed values by ranking *n* observations from largest to smallest with m=1 corresponds to the smallest and m= *n* corresponds to the largest. A good estimator of f(x; *b*; *c*) is as follows (Chow et al, 1988; Wilson, 1983; Linsley et al, 1958; Loveday, 1980):

$$F(x,a,b,c) = \frac{m-0.375}{n+0.25} \quad (10)$$

where, m is the ranking value and n is the number of observations. In order to compute *b* and *c*, first, stability were ranked from the largest to the smallest, ranking values were assigned and functions are computed. Table 2 presents the stability of the electrodes, ranking values and the probability. From the table it can be seen that the probability ranges from 0.051 to 0.949 with the highest probability for the highest stability of the electrodes and the lowest probability for the lowest stability. These expressions agree with standard statistical explanation of cumulative probability meaning that lower stable electrodes will fail first before higher stable electrodes fail. Plots of linear equations (5a and 5b) were conducted to determine the unknown parameters. Table 3 presents the values of each of the steps for the plots. Figures 9- 13 show the linear regression presentations of the results. Figures 9 and 10 are for three parameters and figures 11- 13 are for two parameters. In recent report (Yeh and Chen, 2009; Oke, 2009b; Oke et al, 2009b; Razali et al, 2009), it was suggested that non-linear regression method should be the first choice in determining design parameters for environmental pollution control devices. The values of 'a'; 'b' and 'c' were determined based on these reports as follows:

$$-\log_e\left[1-F(x)\right] = \left(\frac{1}{b}\right)^{c}\left(x-a\right)^{c} \quad (11)$$

$$-\log_e\left[1-F(x)\right] = \left(x\right)^{c}\left(\frac{1}{b}\right)^{c} \quad (12)$$

Table 2 Stability of the electrodes in synthetic wastewater after 24 hours of electrochemical process, rank and probability function

Particle size (μm)	Percentage binder (%)	Compaction pressure (MN/m^2)	Experimental Stability	Ranking values (m)	Estimators $f(x, a, b, c) = \frac{m-0.375}{n+0.25}$
45	8	110	93.4	12	0.949
63	8	110	92.7	11	0.867
45	9	100	92.4	10	0.786
150	8	110	91.3	9	0.704
45	10	90	91.1	8	0.622
63	9	100	90.6	7	0.541
150	9	100	89.6	6	0.459
245	8	110	89.1	5	0.378
63	10	90	88.6	4	0.296
245	9	100	87.7	3	0.214
150	10	90	86.8	2	0.133
245	10	90	84.2	1	0.051

Table 3 Stability of the electrodes in synthetic wastewater after 24 hours, rank, probability function and double logarithms

Experimental Stability	Ranking values (m)	Estimators $f(x, a, b, c) = \frac{m - 0.375}{n + 0.25}$	1-f(x)	$Y_t = (1-f(x))^{-1}$	$Y = Log_e Y_t$	$Log_e Y$
93.4	12	0.949	0.051	19.600	2.976	1.090
92.7	11	0.867	0.133	7.538	2.020	0.703
92.4	10	0.786	0.214	4.667	1.540	0.432
91.3	9	0.704	0.296	3.379	1.218	0.197
91.1	8	0.622	0.378	2.649	0.974	-0.026
90.6	7	0.541	0.459	2.178	0.778	-0.251
89.6	6	0.459	0.541	1.849	0.615	-0.487
89.1	5	0.378	0.622	1.607	0.474	-0.746
88.6	4	0.296	0.704	1.420	0.351	-1.047
87.7	3	0.214	0.786	1.273	0.241	-1.422
86.8	2	0.133	0.867	1.153	0.142	-1.950
84.2	1	0.051	0.949	1.054	0.052	-2.949

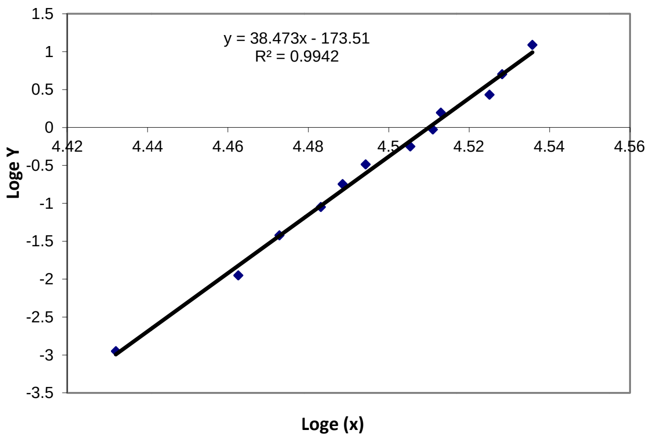

Figure 9. Linear regression of three parameters with variable values of "a"

Figures 13(a to d) present non-linear forms of the Weibull distribution. From the figure (Fig. 13 a) the values of 'a' ; 'b' and 'c'(determined by using Newton's methods, Stroud, 1990) are 0; 91.021 and 38.437 with R^2 equal to 0.9945 and 50.0; 41.020 and 16.792 with R^2 equal to 0. 9937 for two-parameter and three-parameter models respectively.

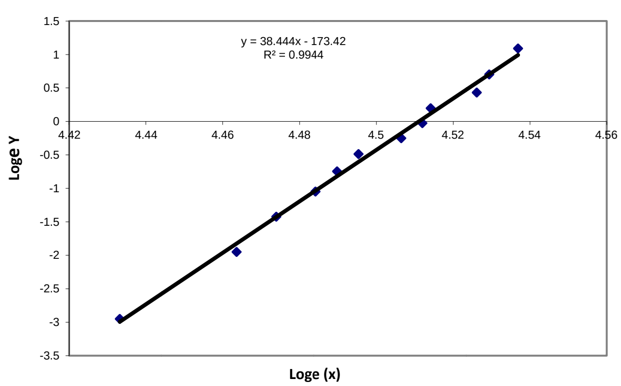

Figure 10. Linear regression of three parameters with a fixed value of "a"

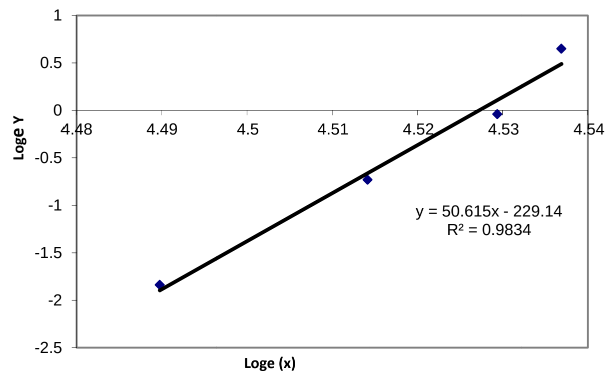

Figure 11. Linear regression of two parameters at 110 MN/ m^2 compaction pressure

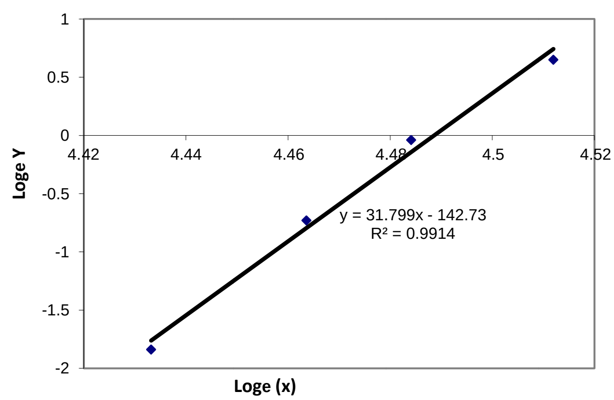

Figure 12. Linear regression of two parameters at 90 MN/ m^2 compaction pressure

Figure 13e presents experimental results and theoretical results (non-linear and linear methods). From this figure (Fig. 13e) it can be seen that these results agree with literature on relationship between linear and non-linear methods. Table 4 shows the values of c, b and correlation coefficients (R^2) with detail explanation.

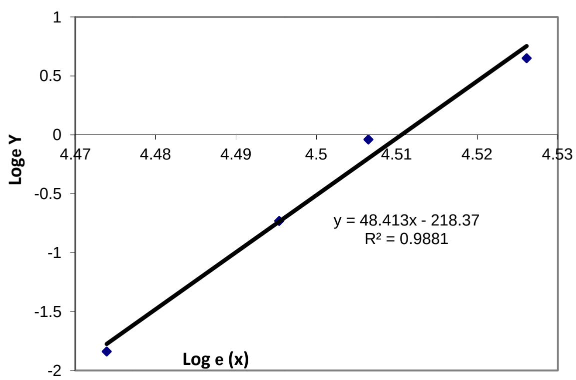

Figure 13. Linear regression of two parameter at 100 MN/ m^2 compaction pressure

From the table the values of c range from 31.80 to 50.62 with an average, standard deviation and coefficient of variation of 41.55, 6.98 and 16.80 respectively. The result indicates that the material tends to be unstable with higher probability for every unit increase in particle size.

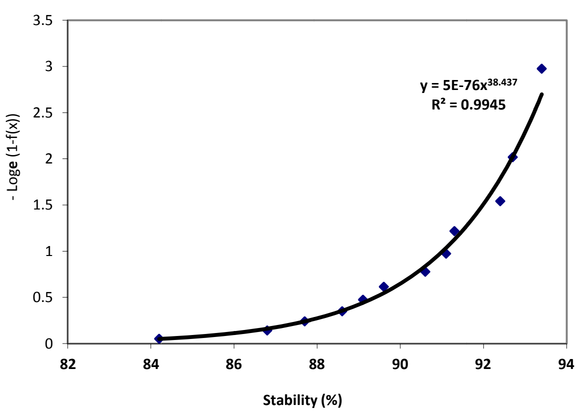

Figure 13(a). Non-linear regression of the two parameters

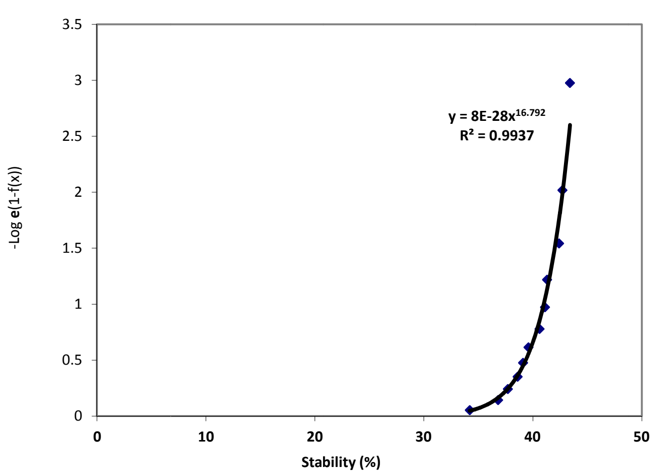

Figure 13(b) Non-linear regression of the three parameters

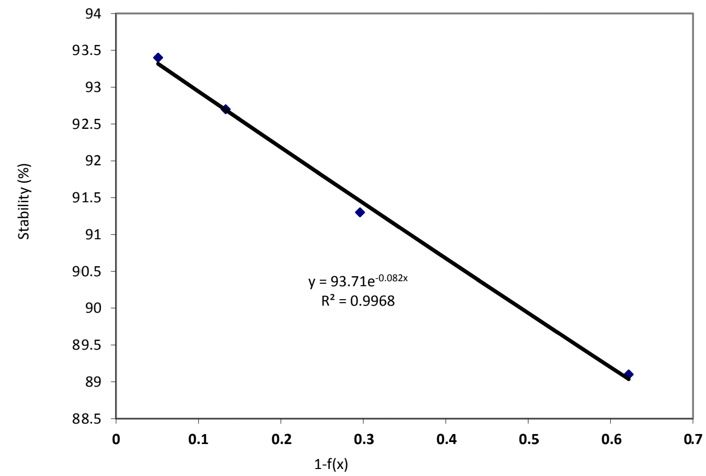

Figure 13(c). Non-linear regression of the parameters at 110 MN/ m^2 compaction pressure

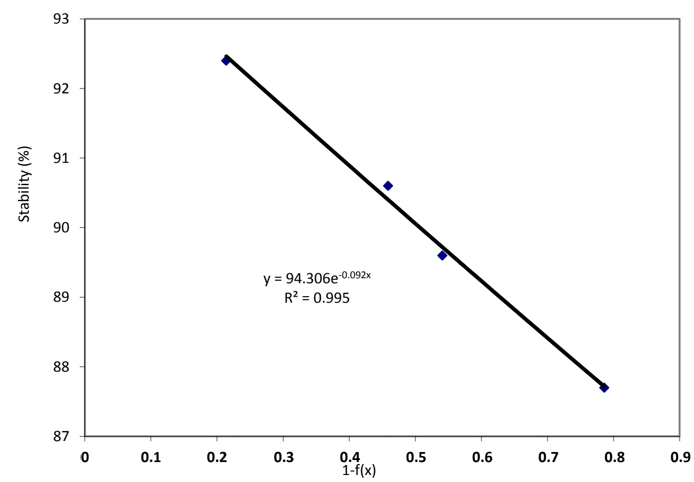

Figure 13(d). Non-linear regression of the parameters at 100 MN/ m^2 compaction pressure

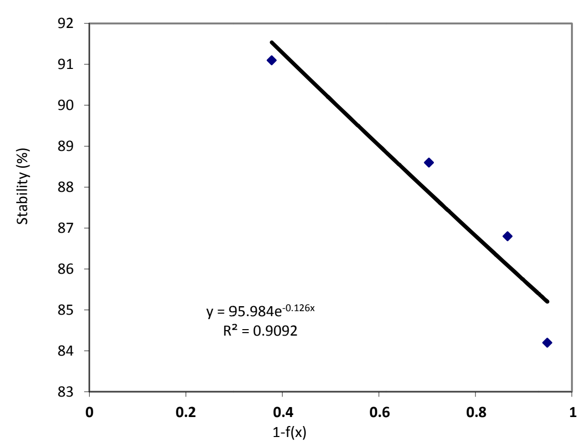

Figure 13(e). Non-linear regression of the parameters at 90 MN/ m^2 compaction pressure

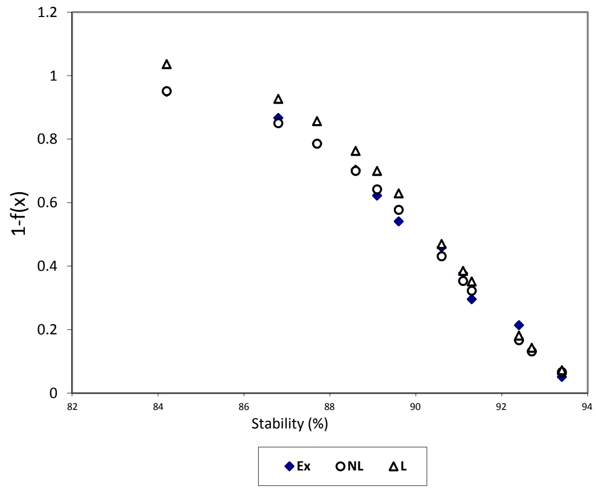

Figure 13(f). Relationship between experimental, linear and non-linear regression results for 2 parameter model

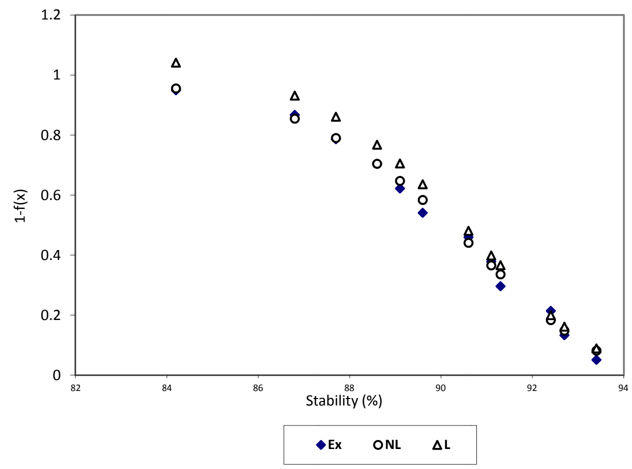

Figure 13(g). Relationship between experimental, linear and non-linear regression results for 3 parameter model

The scale parameter *b* measures the spread in the distribution of data. It has an average, standard deviation and coefficient of variation of 90.88, 1.113 and 1.225 % respectively. The lowest values of b occurs at 90 MN/ m^2 and the highest value at 110 MN/ m^2 indicating that compaction pressure increases the distribution of the stability of carbon-resin electrodes. c is the shape parameter. It is well known that $c < 1$ indicates that the material has a decreasing stability rate. Similarly a $c = 0$ indicates constant failure rate and a $c > 1$ indicates an increasing stability rate. The highest value of c occurs at 110 MN/ m^2 compaction pressure and the least value occurs at 90 MN/ m^2 . With value of c>1 indicates that stability of carbon-resin electrodes increases with compaction pressure and that total collapse or failure of carbon resin electrodes at these compaction pressures is not possible. Reliability of the electrodes was computed using the values of b and c obtained from experimental data and

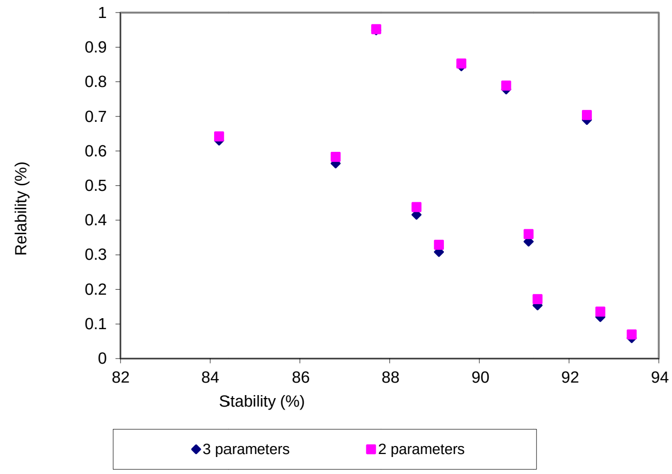

Figure 14. Reliability and stability

equations (3) and (4). The plot of $R(x; b; c)$ is shown in Figure 14. The reliability trend in Figure 14 shows no particular pattern.

In structural engineering, limit state equation for the strength (g(x)) in pure bending can be expressed as

$$g(x) = 1.0 - \frac{M_u}{M_n} \tag{13}$$

It is well known that limit state equation plays an important role in evaluating reliability. It represents the boundary between the safe and unsafe regions and a state beyond which a structure can no longer fulfill the function for which it was designed. A limit state equation can be an explicit or implicit function of the basic random variables and can be linear or nonlinear. Reliability estimation using explicit limit state functions has been discussed in literature such as Cruse et al (1988); Ayyub and Haldar (1984); Haldar and Mahadevan (2000). Haldar and Mahadevan (2000) discussed reliability evaluation techniques for implicit limit state functions. Adopting limit state equation for stability of the electrodes gives:

$$g(x) = 1 - \frac{x_i}{x_n} \tag{14}$$

The limit state equation for stability of electrodes was adopted and applied to the results to establish safe and unsafe region for the stability, particle size, compaction pressure and percentage of the binder. Table 5 and Figure 15 present the values for limit state. From the figure it can be seen that unsafe region is above the critical line, while safe region is below the line (Figure 15a). Figures 16, 17 and 18 show relationship between g(x), particle sizes, compaction pressure and percentage of the binder used.

Table 4 Values of c, b and correlation coefficient

Parameters	c	b	R^2	Explanations
Three variables Weibull distribution	38.47	90.91	0.994	at variable 'a'
	38.44	91.05	0.994	at constant 'a' similar to two variable distribution
Two variables Weibull distribution at fixed compaction pressure	50.62	92.49	0.983	at 110 MN/ m^2 compaction pressure
	31.80	88.99	0.991	at 90 MN/ m^2 compaction pressure
	48.41	90.97	0.988	at 100 MN/ m^2 compaction pressure

Table 5 Values of limit states of the electrodes

Particle size (μm)	Percentage binder (%)	Compaction pressure (MN/m^2)	Experimental Stability	Limit State
45	8	110	93.4	0.0659
63	8	110	92.7	0.0729
45	9	100	92.4	0.0759
150	8	110	91.3	0.0869
45	10	90	91.1	0.0889
63	9	100	90.6	0.0939
150	9	100	89.6	0.1039
245	8	110	89.1	0.1089
63	10	90	88.6	0.1139
245	9	100	87.7	0.1229
150	10	90	86.8	0.1319
245	10	90	84.2	0.1579

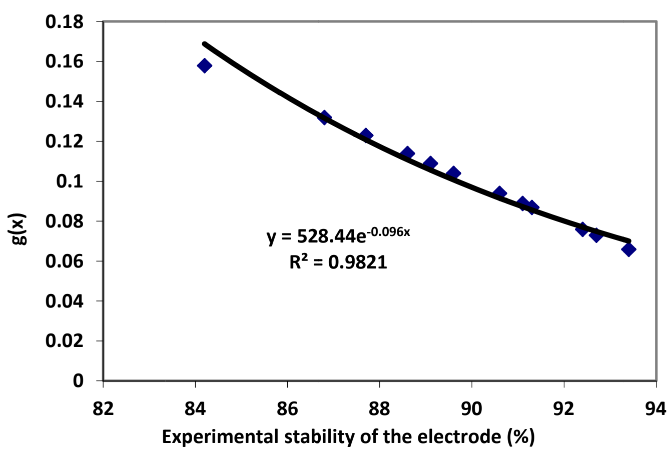

Figure 15 Relationship between experimental stability and limit state

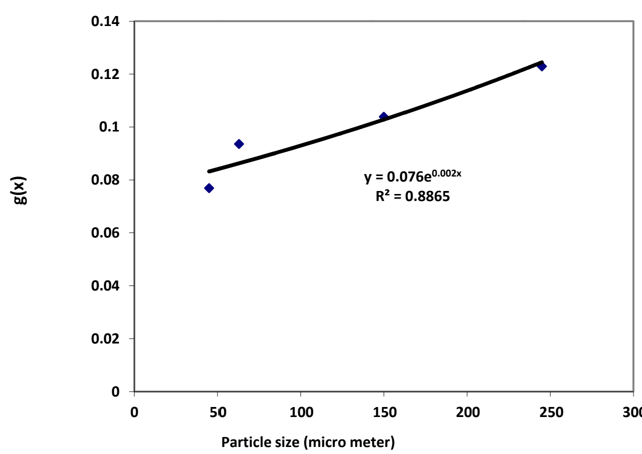

Figure 16 Relationship between particle sizes and limit state

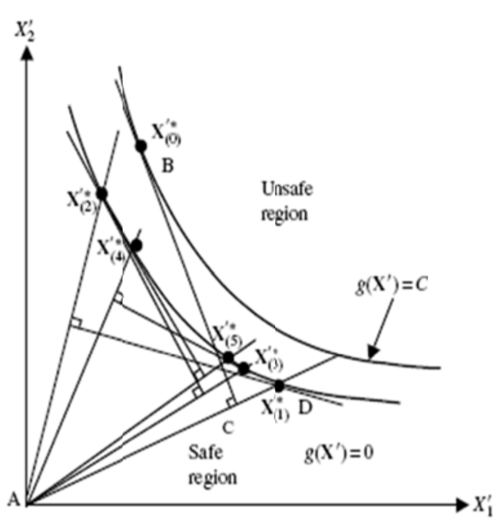

Figure 15a Limit state concept (Haldar and Mahadevan, 2000)

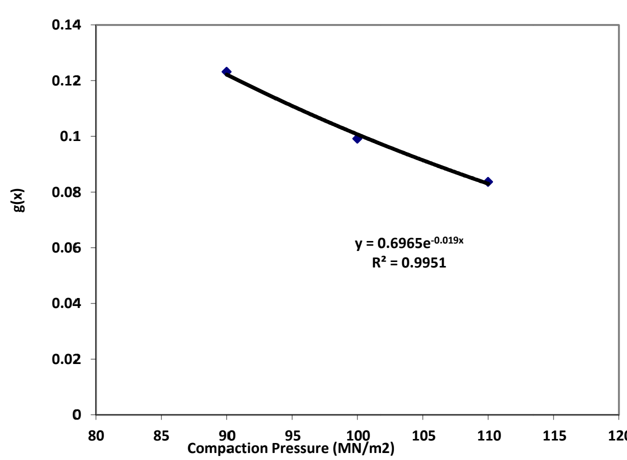

Figure 17 Relationship between Compaction pressure and limit state

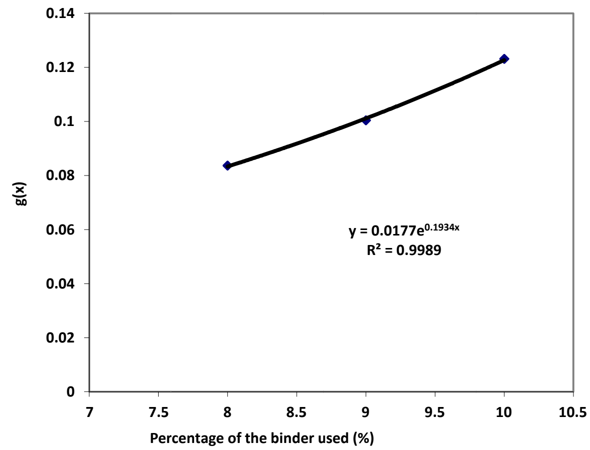

Figure 18 Relationship between percentage of binder and limit state

1.1. Statistical evaluation of the distribution models

The total error, which is the sum of the squares of the errors between the obtained values and the predicted values, can be interpreted as a measure of variation in the values predicted unexplained by the values obtained data (Oke, 2007;2008; Babatola et al, 2008; Gullemo et al, 1999). The lower the value of total error the higher the accuracy, validity and good fitness of the method. Total error (Err^2) can be computed using equation (15):

$$Err^2 = \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2 \qquad (15)$$

Table 6 shows the computation of total error for each of the methods. The total errors were 8.321 and 0.606 for three and two parameter Weibull distributions respectively. These results indicate that two parameter Weibull technique has lower error than three parameter Weibull technique. This can be attributed to the fact that the three parameter Weibull technique was developed using mathematical based expression and were accepted because of its application.

The coefficient of determination (CD) can be interpreted as the proportion of expected data variation that can be explained by the obtained data. Higher values of CD indicate higher accuracy, validity and good fitness of the method. CD can be expressed as follows:

$$CD = \frac{\sum_{i=1}^{n}\left(Y_{obsi} - \overline{Y_{cali}}\right)^2 - \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2}{\sum_{i=1}^{n}\left(Y_{obsi} - \overline{Y_{cali}}\right)^2} \qquad (16)$$

Like total error, CD values are 0.998 and 1.000 for three and two parameter Weibull techniques respectively (Table 6). These results indicate that two parameter Weibull technique has a higher correlation expression than three parameter Weibull technique. The lowest CD in three parameters Weibull technique can be attributed to linearise an exponential equation, coupled with three unknowns in the equation and truncation of the values.

The model selection criterion (MSC) is interpreted as the proportion of expected data variation that can be explained by the obtained data. Like CD the higher the value of MSC, the higher the accuracy, validity and the good fitness of the method. MSC can be computed using equation (17) as follows:

$$MSC = \ln\frac{\sum_{i=1}^{n}\left(Y_{obsi} - \overline{Y_{obs}}\right)^2}{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2} - \frac{2p}{n} \qquad (17)$$

MSC values are 5.739 and 11.496 for three and two parameter Weibull techniques respectively (Table 6). These results indicate that two parameter Weibull technique has the higher MSC than three parameter Weibull technique, which indicate that two parameter Weibull technique should be the first choice in carbon resin electrodes distribution analysis.

2. CONCLUSIONS

Based on the strength of this study, it can be concluded that:

i. stability of carbon resin electrodes depends on compaction pressure, particle size and percentage of binder
ii. carbon resin electrodes are stable at high percentage of binder and compaction pressure and low carbon particle size
iii. statistical evaluation indicated that the two-parameter Weibull distribution is the best for describing the stability of carbon resin electrodes because of its lower total error, and higher CD and MSC.

Table 6 Values of total error, CD and MSC

Two parameters technique							Three parameters technique						
Error			CD		MSC		Error			CD		MSC	
Y_{obs}	Y_{cal}	Err^2	Y_{obs}-mean	Y_{obs}-Y_{cal}	Y_{obs}-mean	Y_{obs}-Y_{cal}	Y_{obs}	Y_{cal}	Err^2	Y_{obs}-mean	Y_{obs}-Y_{cal}	Y_{obs}-mean	Y_{obs}-Y_{cal}
93.4	93.6	0.052003	13.020069	0.052003	13.02007	0.052003	93.4	90.8	7.011597	13.020069	7.010597	13.02007	7.010597
92.7	92.7	9.5E-05	8593.29	9.5E-05	8593.29	9.5E-05	92.7	89.8	8.122826	8.4584028	8.122826	8.458403	8.122826
92.4	92.0	0.130206	8537.76	0.130206	8537.76	0.130206	92.4	89.2	10.09052	6.8034028	10.09052	6.803403	10.09052
91.3	91.5	0.031776	8335.69	0.031776	8335.69	0.031776	91.3	88.7	6.844997	2.2750694	6.844997	2.275069	6.844997
91.1	90.9	0.022752	8299.21	0.022752	8299.21	0.022752	91.1	88.2	8.558265	8299.21	8.558265	8299.21	8.558265
90.6	90.4	0.032971	8208.36	0.032971	8208.36	0.032971	90.6	87.7	8.62134	8208.36	8.62134	8208.36	8.62134

89.6	89.9	0.070258	8028.16	0.070258	8028.16	0.070258	89.6	87.1	6.094796	8028.16	6.094796	8028.16	6.094796
89.1	89.3	0.026137	7938.81	0.026137	7938.81	0.026137	89.1	86.6	6.499924	7938.81	6.499924	7938.81	6.499924
88.6	88.6	0.001189	7849.96	0.001189	7849.96	0.001189	88.6	85.9	7.395881	1.4200694	7.395881	1.420069	7.395881
87.7	87.7	3.37E-05	7691.29	3.37E-05	7691.29	3.37E-05	87.7	85.1	7.006899	7691.29	7.006899	7691.29	7.006899
86.8	86.5	0.084408	7534.24	0.084408	7534.24	0.084408	86.8	83.9	8.402463	7534.24	8.402463	7534.24	8.402463
84.2	84.3	0.008173	7089.64	0.008173	7089.64	0.008173	84.2	81.8	5.930074	7089.64	5.930074	7089.64	5.930074
89.8	89.8	0.0	7343.3		7343.3		89.8	87.1	7.5				
Sum Err		0.367387	88119.43	0.460002	88119.43	0.460002	Sum Err		69.23914	54821.687	90.57858	54821.69	90.57858
Err =0.606			CD=1.000		MSC= 11.496		Err= 8.321			CD=0.998		MSC= 5.739	

LIST OF SYMBOLS

a:	location parameter
b:	scale parameter
c:	shape parameter
$f(x; b; c)$	distribution function
$R(x; b; c)$	reliability of the function
n	observation number
m	ranking value
x	stability of the electrode (%),
ΔM_{sample}	loss in the weight of the electrode at time t after electrochemical treatment (g),
M_{blank}	weight of the electrode in a blank solution at time t after electrochemical treatment (g) and
$M_{initial}$	weight of the electrode in the beginning of electrochemical treatment (g).
Y_{cali}	expected values of each fitting procedure
Y_{obs}	average of observed (experimental) values
Y_{obsi}	observed (experimental) values
MSC	model selection criterion
n	number of data points
CD	coefficient of determination
M_u	unfactored applied moment
M_n	nominal flexural strength of the member
x_i	experimental stability (%)
x_n	nominal stability of the electrode (99.99%).
APHA	American Public Health Association
EX	Experimental
NL	Non-linear
L	Linear

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