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Full Paper

CORRELATIONS FOR NUSSELT NUMBER IN A STAGGERED CROSS-FLOW TUBE-TYPE HEAT EXCHANGER

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ABSTRACT

Empirical correlations for Nusselt number (Nu) in a staggered multi-row multi-column cross-flow tube-type heat exchanger is presented in this paper. In the experiment, air at ambient temperature was drawn by a centrifugal fan perpendicularly over banks of cylindrical rods arranged in staggered configuration of 5 rows by 4 columns. A test element consisting of a tube of pure copper with length, internal and external diameters of 0.125, 0.0115 and 0.0125 m was heated to a maximum temperature of about 90^0C and inserted into the air stream in the working section. Rate of cooling was measured by thermocouple embedded at the centre, via a digital multimeter which was connected to a computer for monitoring temperature data. A semi-logarithms plot of the data was used to calculate the heat transfer coefficient (h) between the copper element and air, and hence the Nusselt number. The heat transfer coefficient at the centre of each of the four columns at ten different flow rates with throttle valve openings ranging from 10 - 100% were investigated. Results showed that the Nusselt number increased exponentially with increase in air flow rate and also increased in successive columns in the direction of flow at a diminishing rate. Correlations of Nusselt number with Reynolds number (Re) were developed for preliminary design and performance assessment of staggered multi-row multi-column cross-flow tube-type heat exchanger.

KEYWORDS: Staggered cross-flow, heat exchanger, Nusselt number, Reynolds number

Nomenclature:

$\dot{q}$ rate of heat transfer to air (J/s)
μ viscosity of air (kg/ms)
A surface area of element (m^2)
A_1 effective surface area of element (m^2)
c specific heat of copper element (J/kg ^{0}C)
c_p specific heat of air at constant pressure (J/Kg ^{0}C)
D outside diameter of element (m)
d inside diameter of element (m)
h coefficient of heat transfer (J/m^2s ^{0}K)
k thermal conductivity of air (J/ms ^{0}C)
l length of copper element (m)
l_1 effective length of element (m)
M slope of cooling curve
m mass of element (kg)
Nu Nusselt Number
p_A barometric pressure (N/m^2)
Re Reynolds Number
t time (s)
T temperature of element (^{o}K)
T_A temperature of air (^{o}K)
V mean velocity past element (m/s)
ρ density of air (kg/m^3)

1. INTRODUCTION

Heat transfer over a bank of tubes in cross flow is of numerous applications in heat exchanger, such as steam generator in a boiler or air cooling in the coil of an air conditioner. In these applications, one fluid moves over the tubes, while a second fluid at a different temperature passes through the tubes and hence, heat is exchanged between the fluids based on the convective heat transfer coefficient. Heat transfer analysis for different configurations of tube-type cross-flow heat exchangers has been investigated by Kays and London (1964) and McAdams (1954).

For cross-flow heat exchangers, the tubes are either arranged in staggered or aligned configuration. The experiment of Incropera and Dewih (2002) has shown that the flow conditions within the bank are dominated by boundary layer separation effects and by wake interactions, which in turn influence the convection heat transfer. Hence, the heat transfer coefficient associated with a tube is determined by its configuration and position of the bank. The heat transfer coefficient of staggered configuration has been reported to be higher than that of aligned (Incropera and Dewih, 2002). Also, for a given configuration, the heat transfer coefficient in the first column from the air intake was found to be lower than that associated with tubes of inner columns. In most configurations, however, heat transfer conditions stabilize, such that little change occurs in the convection coefficient for tube beyond the fourth or fifth column (Incropera and Dewih, 2002; Plint and Partners, 1981).

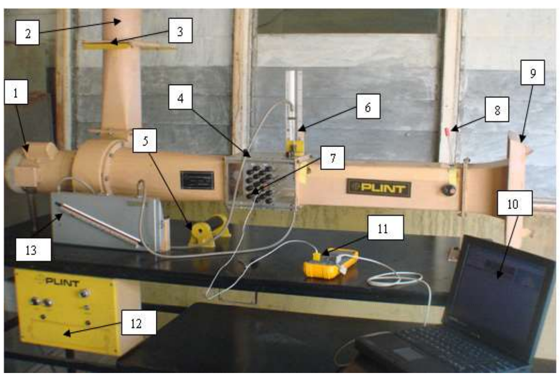
Analysis of the relationships between the heat transfer, flow rate and configuration of the system has been the concern of many researchers over the years. Different analytical models have been developed and applied for modeling this relationship (Zhukauskas, 1972). However, the applicability of these analytical models is limited to a confined range of flow conditions due to the complexity of the relationships. In this sense, therefore, other methods based on numerical techniques have been applied in the analysis such as: artificial neural networks (Pacheco-Vega *et al.*, 2001a; Pacheco-Vega *et al.*, 2001b; Fadare and Fatona, 2008; Fatona, 2008; Islamoglu, 2003), finite element method (Ranganayakulu and Seetharamu, 1999a; Ranganayakulu and Seetharamu, 1999b). However, these numerical methods are approximate solutions, which in most cases, required validation based on empirical data. Different forms of empirical correlations have been proposed for airflow across tube bank with different geometry and configurations (McAdams, 1954; Mwaba et al., 2006).

The aim of this study was to develop empirical correlations for fully developed Nusselt number and Reynolds number of air flowing over a bank of copper tubes at centre of each of the four columns of a staggered cross-flow tube-type heat exchanger.

2. MATERIALS AND METHODS

2.1 Experimentation

The cross-flow heat exchanger apparatus (Plint Engineers, Model TE.93/A, England) was used in this study for purpose of data gathering as shown in Figure 1. The nominal dimensions of the working section and the configuration of the tube bank are shown in Figure 2. Air at ambient temperature was drawn by a centrifugal fan over bank of cylindrical Perspex rods arranged in staggered configuration of 5 rows by 4 columns in Perspex working section. The centrifugal fan was driven by a 1.0 hp induction electric motor at a constant speed of 2,500 rpm. The fan inlet was connected to the downstream of working section and the outlet was connected to a graduated throttle valve by which air velocity through the apparatus was regulated.



1. Fan, 2. Air outlet, 3. Throttle opening, 4. Working section, 5. Electric heater, 6. Total head tube, 7. Test element, 8. Thermometer, 9. Air inflow, 10. Computer, 11. Digital multimeter, 12. Control panel, 13. Inclined water manometer.

Fig. 1: The experimental setup

Detailed experimental procedures, the nominal dimensions, properties of the copper tube and air have been reported elsewhere (Fadare and Fatona, 2008; Fatona, 2008). A thermocouple (K type) of 0.2 mm diameter was imbedded at the centre of the tube to measure its temperature. The thermocouple voltage output is wired to digital multimeter (Mastech®, MAS-345), which was connected to Pentium 4 Laptop computer with DMM View software to record the measured temperature. The ambient temperature of the air was measured with a mercury–in–glass thermometer at the air inlet.

The copper tube was then inserted into the spaces provided in the working section at middle points of each of the 4 columns of the tube bank. For each column position, the rate of cooling of the tube as indicated by a thermocouple embedded at its centre was recorded at the rate of 1 data per second by the computer for 10 different flow rates with throttle openings ranging between 10 - 100%. The velocity distribution upstream and downstream of the tube bank was measured with a total head tube connected to an inclined water manometer.

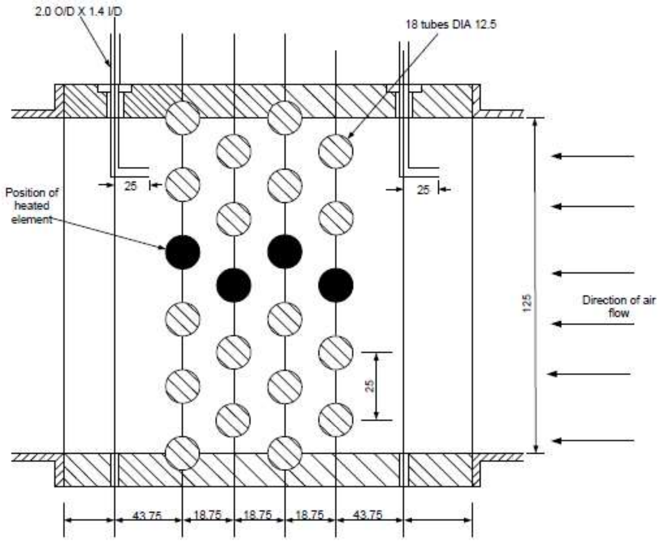


Fig. 2: Nominal dimensions of the working section with staggered tube arrangement

2.2 Data analysis

The air flow rate was measured in terms of the Reynolds number as:

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{1}$$

where (ρ) is the density of air, (V) is the mean velocity, (D) is the outside diameter of tube, and (μ) is the viscosity of air.

For the purpose of estimating the heat transfer coefficient, it was assumed that the whole of the heat lost from the tube is transferred to the air flowing past it. It was also assumed that temperature gradient within the tube thickness was negligible, so that the thermocouple embedded at the centre in the inner diameter gives a true indication of the effective surface temperature of the tube. The rate of heat loss from tube to air is given by:

$$\dot{q} = hA_1(T - T_A) \tag{2}$$

where h is the heat transfer coefficient, (A_1) is the effective surface area of tube, (T) is the temperature of tube, and (T_A) is the temperature of air.

In a period of time dt the temperature drop dT is given as:

$$-\dot{q}dt = mcdT \tag{3}$$

where (m) is the mass of tube, and (c)is the specific heat of copper tube.

Combining equations (2) and (3) and eliminating $\dot{q}$ gives:

$$\frac{-dT}{(T - T_A)} = \frac{hA_1}{mc}dt \tag{4}$$

Integrating equation (4) gives:

$$\log_e(T - T_A) - \log_e(T_0 - T_A) = -\frac{hA_1 t}{mc} \tag{5}$$

where T_0 is the tube temperature at $t = 0$.

Hence, the plot of $\log_e(T-T_A)$ against t yields a straight line of slope:

$$M = -\frac{hA_1}{mc} \tag{6}$$

From this the heat transfer coefficient h can be calculated as:

$$h = -\frac{mc}{A_1}M \quad (7)$$

The fully developed Nusselt number (Nu) is evaluated by

$$Nu = \frac{hD}{k} \quad (8)$$

where (k) is the thermal conductivity of air and (D) is the outside diameter of tube.

Microsoft® Excel 2003 curve fitting algorithm was used to develop the correlations for Nu and Re at the centre of the four columns of the heat exchanger in the form:

$$Nu = A\,\mathrm{Re}^b \quad (9)$$

where A and b are constants.

3. RESULTS AND DISCUSSION

Figures 3 and 4 show typical cooling curve and corresponding semi-log plot obtained for the copper tube positioned at the centre of column 1 with 100% throttle opening with corresponding calculated Reynolds number of 20.9 x 10^3. In Figure 3, it can be seen that temperature of the copper tube decreased exponentially from 90 - 28°C (ambient) within 100 seconds. In Figure 4, the fitted linear model showed that the equation for the cooling curve was $log_e(T-T_A)$ = – 0.0352t + 4.2462 with a high coefficient of determination (R^2-value) of 0.9934. It can be observed that the slope (M) of the linearized cooling curve was equal to -0.0352. The negative sign simply indicated a reduction in temperature of the copper tube with respect to time.

The measured velocity head (H_I), Reynolds number (Re), slope of cooling curve (M) and Nusselt number (Nu) for the copper tube at different column position and throttle openings are given in Table 1. It can be seen that H_I, Re, M and the associated heat transfer property (Nu) increased with increase in throttle opening from 10 – 100% and also increased from column position number 1 – 4. The increase in H_I, Re, M and the associated heat transfer property (Nu) with throttle opening for all the column position can be attributed to the increase in flow rate of the air over the copper tube. As the flow increased from laminar to turbulent flow and thus enhancing the convective heat transfer. The increase in Nu due to the change in position of the copper tube from the outer column 1 to the inner columns can be attributed to the effects of boundary layer separation and wake formation caused by the turbulence of the air over the successive columns of tubes. As the number of column is increased, the boundary layer separation and wake formation tend to reduce after the fourth column. Hence, the heat transfer conditions stabilize, such that little change occurs in the convection coefficient for tube beyond the fourth column.

The plot of Re against Nu for different column positions of the copper tube at different throttle openings (10 – 100%) and the fitted power law model are shown in Figure 5. The Nu increased with increase in Re and increased from column position 1 – 4, though at diminishing rate. Similar observation has been reported by Incropera and Dewih (2002) and Plint and Partners (1981). The correlations between the Nu and Re for the different column positions and the coefficients of determination (R^2-values) between the correlations and experimental data are shown in Table 2. The correlation models gave very high predictive accuracies with R^2-values ranging between 0.9275 and 0.9649.

4. CONCLUSIONS

Empirical correlations for fully developed Nusselt number with Reynolds number for air flowing over a bank of copper tubes at centre of the four columns of a staggered cross-flow tube-type heat exchanger have been developed. The convective heat transfer property measured in terms of Nusselt number increased exponentially with increase in flow rate of the air. Nu increased, though at diminishing rate as the copper tube position is moved from the outer column position 1 to the inner columns. The optimum number of column was found to be four columns for this configuration. The proposed models are useful for preliminary design and performance assessment of staggered multi-row multi-column cross-flow tube-type heat exchanger.

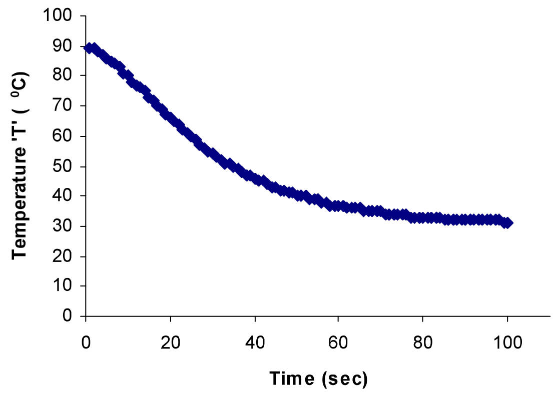


Fig. 3: Typical cooling curve of the heated test element at the centre of column 1 with 100% throttle opening.

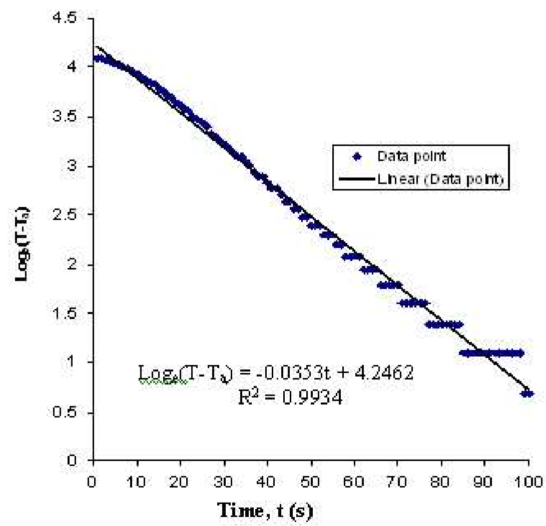


Fig. 4: Typical semi-log plot of the cooling curve of the copper tube at centre of column 1 (100% throttle opening).

REFERENCES

Fadare, D.A. and Fatona, A.S. Artificial neural network modeling of heat transfer in a staggered cross-flow tube-type heat exchanger, Pacific Journal of Science and Technology, 9(2):317-323, 2008.

Fatona, A.S. Heat transfer analysis of air flowing over cylindrical tubes using artificial neural networks. M.Sc. Dissertation of Department of Mechanical Engineering. University of Ibadan, Ibadan, Nigeria, 2008.

Table 1: The velocity head, Reynolds number, gradient of curve and Nusselt number at different throttle openings

% Throttle opening	H_1	Re $X(10^3)$	Column position of test element							
			1		2		3		4	
			M	Nu	M	Nu	M	Nu	M	Nu
10	0.03	3.53	-0.0173	13.4452	-0.0209	16.2430	-0.0225	17.4866	-0.0275	21.3724
20	0.05	4.56	-0.0207	16.0876	-0.0241	18.7301	-0.0282	21.8388	-0.0290	22.5382
30	0.1	6.44	-0.0244	18.9632	-0.0276	21.4501	-0.0321	24.9475	-0.0332	25.8023
40	0.15	7.89	-0.0274	21.2947	-0.0312	24.2410	-0.0352	27.3567	-0.0365	28.3670
50	0.25	10.2	-0.0295	22.9269	-0.0326	25.3360	-0.0368	28.6002	-0.0424	32.9524
60	0.45	13.7	-0.0315	24.4812	-0.0348	27.0458	-0.0384	29.8437	-0.0440	34.1959
70	0.65	16.4	-0.0322	25.0252	-0.0378	29.3774	-0.0417	32.4084	-0.0459	35.6725
80	0.8	18.2	-0.0333	25.8801	-0.0368	28.6002	-0.0433	33.6519	-0.0459	35.6725
90	0.95	19.9	-0.0352	27.3567	-0.0382	29.6882	-0.0425	33.0301	-0.0460	35.7502
100	1.05	20.9	-0.0352	27.3567	-0.0388	30.1545	-0.0417	32.4084	-0.0471	36.6051

H1 = velocity head; Re = Reynolds number; M= slope of cooling curve; Nu = Nusselt number

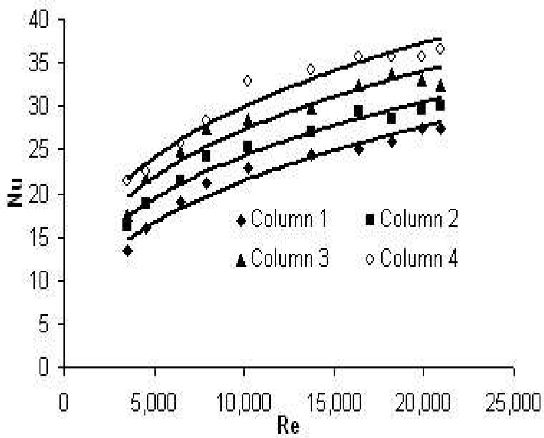


Fig. 5: Plot of Re against Nu for the copper tube at different column positions.

Table 2: Correlation parameters for the copper tube at different column positions

Column position No.	Correlation Model	R^2-value
1	$Nu = 0.7330Re^{0.3667}$	0.9583
2	$Nu = 1.1925Re^{0.3274}$	0.9649
3	$Nu = 1.4340Re^{0.3200}$	0.9275
4	$Nu = 1.6461Re^{0.3152}$	0.9625

Incropera, F.P. and Dewih, D.P. Fundamental of heat and mass transfer, Replika Pub. Ltd, 2002.

Islamoglu, Y. A new approach for the prediction of the heat transfer rate of the wire-on-tube type heat exchanger––use of an artificial neural network model, Applied Thermal Engineering, 23: 243-249, 2003.

Kays, W.M. and London, A.L. Compact heat exchanger, McGraw-Hill, New York, 1964.

McAdams,W.H. Heat transmission, McGraw-Hill, New York, 1954.

Mwaba, M.G., Golriz, M.R. and Gu, J. A semi-empirical correlation for crystallization fouling on heat exchange surfaces, Applied Thermal Engineering, 26(4): 440-447, 2006.

Pacheco-Vega, A., Sen, M., Yang, K.T. and McClain, R.L Neural network analysis of fin-tube refrigerating heat exchanger with limited experimental data, Int. J. Heat and Mass Transfer. 44: 763-770, 2001b.

Pacheco-Vega, A., Sen, M., Yang, K.T. and McClain, R.L. Heat rate predictions in humid air-water heat exchangers using correlations and neural networks, J. Heat Transfer, 123: 348-354, 2001a.

Plint and Partners Ltd. Manual for cross flow heat exchanger-Model TE93/A, England, 1981.

Ranganayakulu, C.H. and Seetharamu, K.N. The combined effects of wall longitudinal heat conduction, inlet fluid flow nonuniformity and temperature nonuniformity in compact tube-fin heat exchangers: a finite element method, Int. J. Heat Mass Transfer, 42(2): 263-273, 1999a.

Ranganayakulu, C.H. and Seetharamu, K.N. The combined effects of longitudinal heat conduction, flow nonuniformity and temperature nonuniformity in crossflow plate-fin heat exchangers, Int. Commun. Heat Mass Transfer, 26(5): 669-678, 1999b.

Zhukauskas, A. Heat transfer from tubes in cross flow, In J.P. Hartnett & T.F. Irvine Jr., Eds., advances in heat transfer, Vol. 8. Academic Press, New York, 1972.