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NEWTON-BASED OPTIMAL POWER FLOW ALGORITHM INCORPORATING SVC AND TCSC

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ABSTRACT

This paper presents the results of study of the power system optimal power flow (OPF) incorporating Static Var Compensator (SVC) and Thyristor Controlled Series Compensator (TCSC). The steady state models of Static Var Compensator (SVC) and Thyristor-Controlled Series Compensator (TCSC) were incorporated into the non-linear equations of the power system networks. The objective function which is the fuel cost was then optimized subject to the equality constraints of the real and reactive power balance and the inequality constraints of the bus voltages. A Newton based optimal power flow algorithm was developed and this was applied to the 5-bus and 26-bus systems. Results of the optimal power flow solution with and without these Flexible Alternating Current Transmission System (FACTS) devices were presented to show the effectiveness of these devices on the power networks performance. The results show that the incorporation of these FACTS devices into the power system network can strengthen the network and increases its transmission capability.

Keywords: *FACTS, TCSC, SVC, OPF, Newton's method of power flow solution.*

1. INTRODUCTION

The expansion and growth of the electric utility industry, and the ongoing deregulation in many countries have introduced numerous changes into a once predictable business of electric power generation and distribution. Although electricity is a highly engineered product, it is increasingly being considered and handled as a commodity (Paserba, 2004). Before deregulation, system control and stability are ensured by the operating engineer. With deregulation, the engineer has no direct control over the generation and loading of the system. Thus, many power system networks are stressed to their limits and suffer frequent system failures. In principle, limitations on power transfer can always be relieved by the addition of new transmission facilities. Alternatively, FACTS controllers can achieve the same objective with no major alterations to system layout (Hingorani and Gyugyi, 2000). The potential benefits brought about by FACTS controllers include reduction of operation and transmission investment cost, increased system security and reliability, increased power transfer capabilities, and an overall enhancement of the quality of the electric energy delivered to customers (Hingorani and Gyugyi, 2000).

Research had been done to investigate the impacts of FACTS devices on the performance of power system and the use of SVC and STATCOM to improve the voltage profile of the network have been studied (Paserba, 2004; Ambriz-Pe´rez, 2000). Likewise, the applications of the TCSC to control the power flow on a transmission line (Gherbi et al., 2009; Ambriz-Pe´rez, 2006; Kazemi and Badrzadeh, 2004) have also been studied. Nevertheless, further study of the impact of FACTS devices on optimal power flow is still needed to a great extent.

Optimal Power Flow (OPF) solutions are used to determine the optimum values of the operating states of a power network by optimizing an objective function while satisfying a set of physical and operational conditions (Dommel and Tinney, 1968; Sun et al., 1984; Ambriz-Pe´rez, 2000). This kind of problem is usually expressed as a static, nonlinear programming problem, with the objective function represented as a set of nonlinear equations and the constraints represented by nonlinear or linear equations (Dommel and Tinney, 1968; Weber, 1997)

The objective of this work is to study the effects of SVC and TCSC in a Newton-based optimal power flow algorithm.

2. PROBLEM FORMULATION FOR OPTIMAL POWER FLOW

The OPF problem can be formulated as follows

$$\left.\begin{array}{ll} \text{Minimize} & f(z) \\ \text{such that} & g(z)=0 \\ & h(z)\leq 0 \end{array}\right\rangle \quad (1)$$

Where

$$z=\left[x^{T} \quad y^{T}\right] \quad (2)$$

$f(z)$ is the objective function to be optimized,
$g(z)$ represents the power balance equation and
$h(z)$ consists of state variable limits.

The elements of x and y are the parameters of a power system network defined in equations (3) and (4).

$$x = \begin{bmatrix} V,\delta & P,V & P,Q \\ \text{at slack} & \text{at all} & \text{at all} \\ \text{bus} & \text{generator} & \text{load} \\ & \text{buses} & \text{buses} \end{bmatrix} \quad (3)$$

$$y = \begin{bmatrix} P,Q & Q,\delta & V,\delta \\ \text{at slack} & \text{at all} & \text{at all} \\ \text{bus} & \text{generator} & \text{load} \\ & \text{bus es} & \text{buses} \end{bmatrix} \quad (4)$$

P and Q are the real and reactive power respectively, while V and δ are the voltage magnitude and phase angle respectively.

3. Objective Function

The aim of the OPF solution is to determine the control settings and system state variables that optimize the value of the objective function. The objective functions that may be considered in OPF include the power generation cost, transmission loss and power generation emission. Power generation cost is the most popular objective function where the thermal generation unit costs are generally represented by a nonlinear, second order function (Sun et al., 1984) as shown in equation (5).

$$F = \sum_{i=1}^{ng} f_i(P_{gi}) = \sum_{i=1}^{ng}\left(a + bP_{gi} + cP_{gi}^2\right) \quad (5)$$

Where a, b and c are the cost coefficients.

4. Equality Constraints

The equality constraints of OPF are the real and reactive power balance equations that are given by equations (6) and (7)

$$\begin{aligned} &V_i \sum_{k=1}^{ng} V_k Y_{ik} \cos\left(\delta_k - \delta_i + \alpha_{ik}\right) + P_{di} - P_{gi} = 0 \\ &P_i(V,\delta) + P_{di} - P_{gi} = 0 \end{aligned} \quad (6)$$

$$\begin{aligned} &V_i \sum_{k=1}^{ng} V_k Y_{ik} \sin\left(\delta_k - \delta_i + \alpha_{ik}\right) + Q_{di} - Q_{gi} = 0 \\ &Q_i(V,\delta) + Q_{di} - Q_{gi} = 0 \end{aligned} \quad (7)$$

Where

V_i and V_k are the voltage magnitudes at buses i and k respectively

δ_i and δ_k are the voltage phase angles at buses i and k respectively

Y_{ik} and θ_{ik} are respectively the magnitude and phase angle of the admittance line between bus i and bus k.

P_{di} and Q_{di} are respectively, the active and reactive load at bus i;

P_{gi} and Q_{gi} are respectively, the scheduled active and reactive power generation at bus i.

nb is the number of bus in the network.

5. Inequality Constraints

The power and voltage variables have upper and lower limits that must be satisfied. These limits reflect the bounds of the operating conditions of the equipment used for power dispatch.

$$\left.\begin{aligned} &P_g^{min} \le P_g \le P_g^{max} \\ &Q_g^{min} \le Q_g \le Q_g^{max} \\ &V^{min} \le V \le V^{max} \end{aligned}\right\rangle \quad (8)$$

If a reactive power limit violation takes place in a generator bus, it changes to a load bus, with associated voltage constraints (Acha et al., 2004).

6. Optimal Power Flow Solution By Newton Method

Solving the optimal power flow problem requires that the equality and inequality constraint equations be augmented and combined with the objective function to obtain an unconstrained optimization problem. This involves the utilization of Lagrangian multipliers and penalty functions (Ambriz-Pe´rez, 1998). The resulting equation (9) is called an Augmented Lagrangian function.

$$L(z,\lambda) = f(z) + \lambda^t g(z) + \Psi(h(z),\mu) \quad (9)$$

Where

$$\Psi(h(z),\mu) = \left\langle \begin{aligned} &\mu\left[h(z) - \overline{h}\right] + 0.5c\left[h(z) - \overline{h}\right]^2 \\ &\quad \text{if } \mu + c\left[h(z) - \overline{h}\right] \ge 0 \\ &\mu\left[h(z) - \underline{h}\right] + 0.5c\left[h(z) - \underline{h}\right]^2 \\ &\quad \text{if } \mu + c\left[h(z) - \underline{h}\right] \le 0, \text{ and} \\ &\quad 0 \quad \text{otherwise} \end{aligned} \right. \quad (10)$$

λ and μ are Lagrange multiplier vectors for equality and inequality constraints respectively,

$\psi(h(z), \mu)$ is a generic function for handling inequality constraints.

$\overline{h}$ and $\underline{h}$ are the upper and lower limits on the variables.

If $\psi(h(z), \mu)$ is neglected in equation (9), and $f(z)$ and $g(z)$ are written explicitly, equation (9) becomes

$$\begin{aligned} L(z,\lambda) = &\sum_{i=1}^{ng}\left(a + bP_{gi} + cP_{gi}^2\right) \\ &+ \sum_{i=1}^{nb} \lambda_{pi}(P_i(V,\delta) + P_{di} - P_{gi}) \\ &+ \sum_{i=1}^{nb} \lambda_{qi}(Q_i(V,\delta) + Q_{di} - Q_{gi}) \end{aligned} \quad (11)$$

λ_{pi} and λ_{qi} are the Langrange multiplier relating to the active and the reactive power balance equations.

The optimization process is by finding the first partial derivative of the augmented Langrangian function with respect to z and λ variables and setting the resulting equations to zero to yield equation (12).

$$\left.\begin{aligned} \frac{\partial L(z,\lambda)}{\partial z} &= \nabla_z L(z,\lambda) = 0 \\ \frac{\partial L(z,\lambda)}{\partial \lambda} &= \nabla_\lambda L(z,\lambda) = 0 \end{aligned}\right\rangle \quad (12)$$

The elements of z involved in the partial differentiation are the voltage magnitude, voltage phase angle and the real power generation. Thus, equation (12) consists of a set of non-linear equations which can be solved by the Newton solution method.

If equation (12) is linearized, equation (13) is obtained.

$$[H]\begin{bmatrix} \Delta z \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} \nabla_z L(z,\lambda) \\ \nabla_\lambda L(z,\lambda) \end{bmatrix} \quad (13)$$

or

$$\begin{bmatrix} \Delta z \\ \Delta \lambda \end{bmatrix} = [H]^{-1}\begin{bmatrix} \nabla_z L(z,\lambda) \\ \nabla_\lambda L(z,\lambda) \end{bmatrix} \quad (14)$$

Where

$\begin{bmatrix} \nabla_z L(z,\lambda) \\ \nabla_\lambda L(z,\lambda) \end{bmatrix}$ is the gradient vector

$[H] = \begin{pmatrix} \frac{\partial^2 L(z,\lambda)}{\partial^2 z} & \frac{\partial^2 L(z,\lambda)}{\partial z \partial \lambda} \\ \frac{\partial^2 L(z,\lambda)}{\partial \lambda \partial z} & \frac{\partial^2 L(z,\lambda)}{\partial^2 \lambda} \end{pmatrix}$ is the Hessian matrix

Equation (13) is solved iteratively for Δz and $\Delta\lambda$ until the mismatches are close to zero. z and λ are updated after each solution of equation (14) for Δz and $\Delta\lambda$, as follows.

$$\left.\begin{aligned} z^{new} &= z^{old} - \Delta z \\ \lambda^{new} &= \lambda^{old} - \Delta \lambda \end{aligned}\right\rangle \quad (15)$$

7. Handling Of Inequality Constraints

There are two methods of handling the inequality constraints (i) multiplier method and (ii) penalty method. The inequality constraints are handled in the OPF formulation by means of the multiplier method in this development as oppose to the penalty function method (Luenberger, 1984; Bertsekas, 1982).

When a variable is within limits, the inequality constraint is inactive, but when violation occurs, it becomes active. The inequality constraints, when made active, are changed to equality constraints. This has the effect of bringing back the variables that are outside limits within limit. In the multiplier method, the Lagrangian function is further augmented by a generic term shown in equation (10). This equation will only be included in the augmented Lagrangian if a variable is not within limit. After the Lagrangian function has been augmented by the term given in equation (10), the first and second partial derivatives corresponding to the inequality constraint are included into the gradient vector and Hessian matrix respectively. The linearized equation (13) is again solved and the variable is updated using equation (15).

At a given iteration, *(i +1)*, the multipliers are updated according to the following criteria:

$$\mu_k^{(i+1)} = \left\langle \begin{array}{l} \mu_k^i\left[h_k(z^{(i)}) - \overline{h}_k\right] + c^{(i)}\left[h_k(z^{(i)}) - \overline{h}_k\right]^2 \\ \quad \text{if } \mu_k^i + c^{(i)}\left[h_k(z^{(i)}) - \overline{h}_k\right] \geq 0 \\ \\ \mu_k^i\left[h_k(z^{(i)}) - \underline{h}_k\right] + c^{(i)}\left[h_k(z^{(i)}) - \underline{h}_k\right]^2 \\ \quad \text{if } \mu_k^i + c^{(i)}\left[h_k(z^{(i)}) - \underline{h}_k\right] \leq 0, \text{and} \\ \\ 0 \quad \text{otherwise} \end{array} \right.$$

(16)

After moving a variable to one of its limits, the algorithm holds it there for as long as it is required, otherwise the variable is freed.

8. Incorporating FACTS Devices

As stated earlier, FACTS devices are used to strengthen the power system network. In this section, the inclusion of the FACTS devices such as the SVC and TCSC will be discussed

8.1 Incorporation of SVC

The SVC is always shunt connected to the bus as shown in figure 1 (Ambriz-Pe´rez, 2000) below in other to control the voltage to the required magnitude.

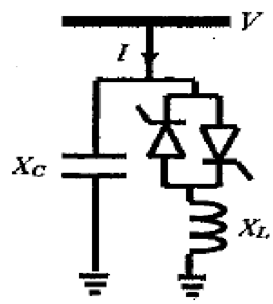

Figure 1 Static Var Compensator Structure

The SVC consists of an inductor, capacitor and thyristor. By controlling the firing angle of the thyristor, the device is able to control the capacitive reactance connected to the bus.

The inclusion of SVC into the optimal power flow problem only introduces an additional variable which is the SVC firing angle. This SVC state variable is combined with the network state variables for a unified, optimal solution using Newton's method (Ambriz-Pe´rez, 1998). Incorporating SVC will introduce one extra equation into the augmented Lagrangian. This equation is given below

$$L_{SVC}(z,\lambda) = \frac{-\lambda_{qi} V_i}{X_C X_L}\left(\begin{array}{l} X_L - \\ \frac{X_C}{\pi}\left\{2(\pi - \alpha_{SVC}) + \sin(2\alpha_{SVC})\right\} \end{array} \right) \quad (17)$$

Where

V_i and λ_i are the bus voltage magnitude and lambda relating to reactive power balance equation of the bus at which the SVC is connected respectively.

X_L and X_C are the inductive and capacitive reactance respectively.

α_{SVC} is the thyristor firing angle.

The first and second partial derivatives of equation (17) are assembled to form the gradient vector and Hessian matrix respectively. The elements of the matrix corresponding to the V_i and λ_{qi} is added to that for the base case. The firing angle creates additional variable in the OPF formulation. For each SVC present in the network, gradient vector is augmented by one row and Hessian matrix is augmented by one row and one column. After augmenting, equation (13) is solved and the variables involved are updated using equation (15). The inequality constraint on the firing angle is also handled using equation (10). Initial value for the firing angle is selected between 90^0and180^0 that are the limits allowed.

8.2 Incorporation of TCSC

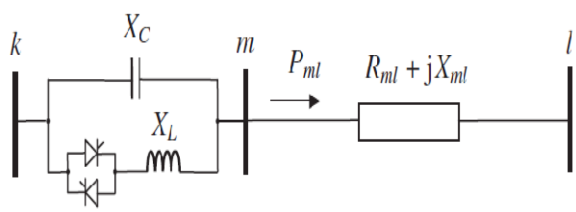

Figure 2 A TCSC compensated transmission line

The TCSC as shown in the figure 2 (Ambriz-Pe´rez et al., 2006) above is usually connected in series with the transmission line to directly control line currents. The active power flow P_{ml} is controlled by the TCSC connected between buses k and l.

The operating condition is represented as an equality constraints which is stated as follows

$$P_{ml} - P_{fixed} = 0 \qquad (18)$$

Where P_{ml} is the active power flow on line m-l, P_{fixed} is the expected active power flow on the line.

The Lagrange function, L, of branch k-l may be expressed by

$$\begin{aligned} L = &\lambda_{pk}\left(P_k + P_{dk} - P_{gk}\right) + \lambda_{qk}\left(Q_k + Q_{dk} - Q_{gk}\right) \\ &+ \lambda_{pm}\left(P_m + P_{dm} - P_{gm}\right) + \lambda_{qm}\left(Q_m + Q_{dm} - Q_{gm}\right) \\ &+ \lambda_{ml}\left(P_{ml} - P_{fixed}\right) \end{aligned} \qquad (19)$$

Where

$$\left.\begin{aligned} P_k &= V_k V_m B_{km} \sin(\delta_k - \delta_m) \\ Q_k &= V_k^2 B_{kk} \cos(\delta_k - \delta_m) \\ B_{kk} &= -B_{km} = B_{TCSC} \\ B_{TCSC} &= 1 / X_{TCSC} \end{aligned}\right\} \qquad (20)$$

To obtain the equations for parameters at bus m, subscripts k and m are exchanged in equation (20).

The inclusion of TCSC into the optimal power flow problem introduces two additional variables which are the TCSC reactance (X_{tcsc}) and the lambda (λ_{ml}) relating to the equality constraints of the active power flow to be fixed. Additional function that the TCSC will introduce into equation (11) is given in equation (19). The first and second partial derivatives with respect to V, δ, λ_p and λ_q of this equation are assembled to form the gradient vector and Hessian matrix respectively. These partial derivatives are added to that for the base case. For each TCSC present in the network, gradient vector is augmented by two rows while the Hessian matrix is augmented by two rows and two columns. After the assembling of the new gradient vector and Hessian matrix, equation (13) is solved and the variables involved are updated using equation (15)

The TCSC reactance initial value can be between $\pm 0.5X_{tl}$ where X_{tl} is the reactance of the compensated line. λ_{ml} is initially set is zero. The limit considered for the TCSC reactance is $-0.5X_{tl}$ to $+0.5X_{tl}$.

9. Test Cases

A MATLAB program was developed to solve the OPF problem with SVC and TCSC incorporated. The program has been tested on a 5-bus system presented in (Acha et al., 2004) and the result obtained were comparable with the one presented by (Acha et al., 2004). The program was further tested on a 26-bus system that contains 6 generators (Sadaat, 2002). The voltage magnitude limits on all buses were set between 0.95 and 1.05pu. The data for the 6 generators are shown in Table 1.

Table 1: Generator bus data of the 26 bus system

Bus no	Pgmax (pu)	Pgmin (pu)	a ($/hr)	b ($/MWH)	c ($/MW^{2}hr)
1	5.0	1.0	240	7.00	0.007
2	2.0	0.5	220	10.0	0.0095
3	3.0	0.8	200	8.50	0.009
4	1.5	0.5	220	11.0	0.009
5	2.0	0.5	200	10.5	0.008
26	1.2	0.5	190	12.0	0.0075

Results are presented for three different cases; case 1 without FACTS devices, case 2 with SVC at bus 24 and case 3 with SVC at bus 25. The SVC was used on these buses to control the voltage magnitude to 1pu.

Table 2 Comparison of computational results of each case

Case no	Total active generation (MW)	Total fuel cost ($/hr)	Total active loss (MW)	Total reactive generation (MVAR)
1	1274.24	15427.0	11.2407	597.888
2	1273.83	15421.4	10.8264	564.838
3	1274.05	15424.5	11.0528	581.260

From the results shown on Table 2 above it can be seen that the introduction of SVC into the network has caused a reduction of reactive power generation which affected the distribution of reactive power throughout the network. This in turn resulted in reduction of the transmission loss and active power generation cost.

The bus voltage magnitude for each case is shown in Figure 3. Since buses 19, 21 and 22 are directly connected to bus 24, their voltage magnitudes improved by the injection of 30.620 MVAr at bus 24 for case 2. Bus 25 is also connected to buses 11, 19 and 23, so the voltage magnitude at these buses were improved by the injection of 15.505 MVAR reactive power at bus 25 for case 3.

To illustrate the effect of TCSC on the system, three cases are also presented: case 1 without FACTS, case 2 with TCSC on line 13-14 to fix the active power flow at 73.12 MW and case 3 with TCSC on the same line to fix the active power flow at 93.12 MW respectively. The base case active power flow on this line was 83.12 MW.

Table 3 shows the results for total active generation, total generation cost, total active loss and total reactive generation for each case. The insertion of TCSC changes the network power flows and may result in increase in the system transmission losses.

Table 4 shows the impact of TCSC on lines connected to bus 13. As expected, the active power flow towards this bus decreased

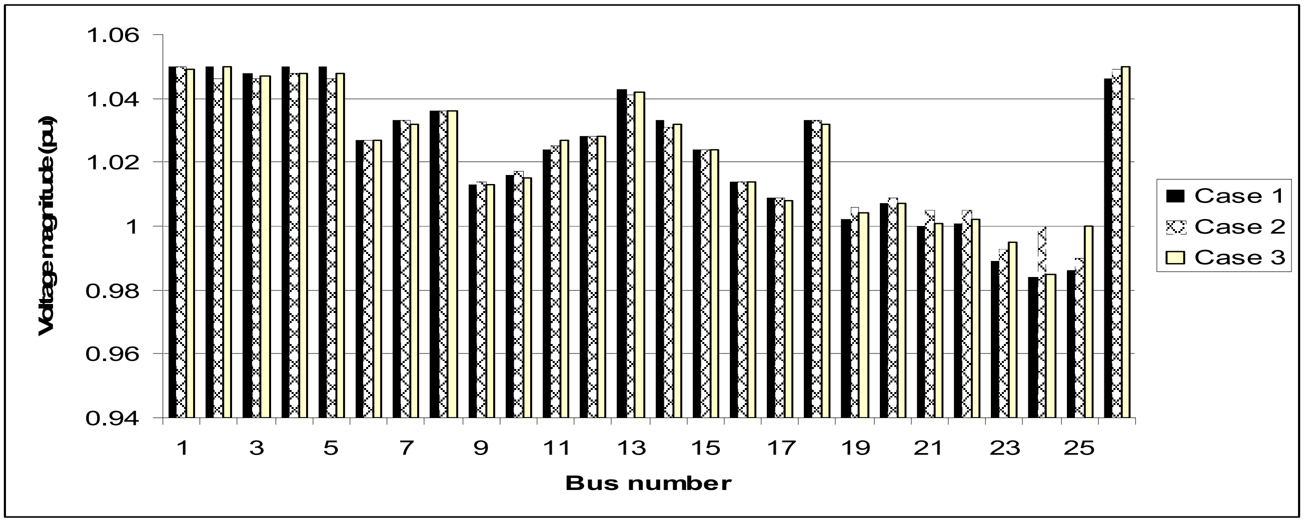

Figure 3 Comparison of computational results of each case

Table 3 Comparison of computational result of each case

Case no	Total active generation (MW)	Total generation cost ($/hr)	Total active loss (MW)
1	1274.24	15427.0	11.2407
2	1274.23	15426.8	11.2282
3	1274.27	15427.3	11.2653

for case 2 and increased for case 3 while active power flow leaving bus 13 increased for case 2 and decreased for case 3. This is so because changing the active power flow by the TCSC will affect the line reactance and line current. Thus line current and line flows of other lines will also be affected.

Table 4 Comparison of computational results of each case

Transmission Line From	to	Case 1 Active power flow (MW)	Case 2 Active power flow (MW)	Case 3 Active power flow (MW)
2	13	21.015	15.257	22.595
3	13	213.498	211.548	216.180
15	13	-61.225	-65.697	-57.252
16	13	-57.728	-59.843	-56.029

NB: The negative sign mean that the active power flow is towards the sending bus

10. Conclusion

An optimal power flow algorithm based on the Newton solution technique that includes SVC and TCSC has been developed and presented in this work. The 5-bus and 26-bus system have been tested on the algorithm. Comparison of results obtained for the 26-bus with and without FACTS have been presented. Based on the results presented, it was demonstrated that FACTS devices can be used as control devices in the power system network to improve system performance. The SVC has been used to bring the bus voltage magnitude up to the required level by injecting reactive power into the bus and the TCSC has been used to either increase or reduce the line reactance in other to control the line current and active power flow. The generation cost reduced when the SVC was used to improve bus voltage magnitudes.

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