Full Paper

SINGLE BEAM MODEL FOR PREDICTING INTRUSION IN PIPELINES

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ABSTRACT

This paper examined pipeline safety and developed a predictive model for intrusion-induced failures. A mathematical model for predicting intrusion has been formulated. A typical pipeline structure has been idealized as a circular beam resting on an elastic soil foundation. Intrusion was considered to consist of hammering and cutting activities. From the solution obtained for the model a feedback control algorithm has also been propounded which could in turn help in designing surveillance instruments which can assist in mitigating the problem. The feasibility of introducing displacement sensors into a typical pipeline to enable collection of spatially distributed modal response was explored in an example. The idea was that sensors react to intrusive load signals and could be configured to measure physical parameters (displacement, velocity or acceleration response). A point sensor was used to simulate the response in a 2 m segment of the pipe to intrusive drilling force. The response at intervals of 2 s was randomly sampled at 0.00, 0.60, 1.10, 1.40 and 2.00 m and displacements indicated were 0.00, -1.25×10^{-3}, -1.82×10^{-3}, -1.56×10^{-3}, 0.00 m, respectively. A peak displacement of around -1.83×10^{-3} m was obtained at the point of intrusion.

Keywords: *pipeline, intrusion, two-parameter elastic foundation and Timoshenko beam*

1. INTRODUCTION

Pipeline facilities are fundamental requirements in the mining, processing and transportation of liquid and gas hydrocarbons from one point to another, often over large distances. In Nigeria, pipelines have remained central to economic growth. About 10,480,533 metric tons (approximately 12,330,040 m^3) of petroleum products was evacuated mainly through its pipelines in 2002 (NNPC Annual Statistical Bulletin, 2002). In comparison, the United States of America transports around 1,324,800 m^3 daily (about 483,552,000 m^3 per annum) of refined petroleum products from refineries to distribution centers and more than 1.21×10^9 m^3 (about 7.6 billion barrels) of crude oil annually from the oilfields to refineries through pipelines(Shell, U.S.A., 2011).

Pipeline failures are mainly attributable to operational and technical causes such as corrosion of aged sections, pressure surges, construction/earth moving activities around pipeline right of ways(ROW) etc. (Mohitpour *et al.*, 2000; Okoli *et al.*, 2003). However, the major causes of failure have recently shifted to malicious human actions and tampering such as 'hot-tapping' pipelines and siphoning their contents (Ogbeifun, 2007; Walker, 2008; Yo-Essien, 2008). The deliberate human activities are termed intrusion.

Pipeline intrusion results in monumental, but avoidable waste of both human and material resources, as well as other consequential losses. It is reported that between 2005 and 2008, a total of 11,503 failures occurred in NNPC's pipeline network, out of which 11,350 were allegedly due to vandalism (NNPC Annual Statistical Bulletin, 2005 – 2008). Pipeline vandalization has been reported in Texmelucan, Central (BBC News, 20 December, 2010). The challenge to safety and the negative impact of pipeline failures on the economy has prompted the need to review existing pipeline engineering practices.

Pipeline codes and standards currently in use, does not address the contemporary problem of pipeline intrusion. ASME B31 design criteria (commonly employed in Nigeria) do not specify any rule for design against third party intrusions. It provides only for Location Class factors to allow for vulnerability due to pipeline-human interface within a specified limit of pipeline right of ways. The design factor merely increases the wall thickness of the pipeline in heavily populated areas such as cities, road, river and railway crossings, while the normal factors apply in sparsely populated areas.

Several models for predicting failure phenomena such as corrosion flaws and leaks have been developed (Munn, 1992; ASME, 2000 and Akhigbemidu, 2005). The problem of "pipe walking" whereby pipes subjected to internal and external temperature fluctuations, in addition to repeated operational start-up and shut-down procedures which triggers vibrations of the pipes, and thereby leading to finite and irreversible longitudinal extension of the pipe over time has been modelled by Olunloyo *et al.* (2007). Villarraga *et al.* (2004) have also used Euler-Bernoulli beam theory to analyze stresses and displacements in underground pipelines with manufacturing imperfections. However, existing models have not addressed the challenge of pipeline intrusion commonly encountered by pipeline operators in Nigeria and elsewhere. This work is an attempt to use valid engineering and mathematical concepts to model intrusion in pipelines in order to detect its occurrence. The model was used because it lends itself to measurable indices of displacement, velocity and acceleration arising from mechanical third party interference.

2. METHODOLOGY

A review of pipelines as designed and built in Nigeria was undertaken to establish the weaknesses that predispose it to illegal intrusion. The implements and procedures utilized by vandals were studied and their generic force patterns noted. Geometric and mathematical models capable of representing the forces from such malicious intrusions were formulated as well. The structural response to such attacks in terms of displacement, velocity and acceleration were determined. Feedback control algorithms were then proposed from the solutions of the mathematical model which simulated the responses. This could in turn help in designing surveillance instruments which can assist in mitigating the problem. An application example, establishes the feasibility of introducing displacement sensors to collect spatially distributed data from a typical segment of a pipeline undergoing a cutting action.

2.1. Intrusion Prediction Modelling

2.1.1. Description of the Model

A typical pipeline, depending on the terrain of the right of way and other design considerations, consists of the steel pipes covered externally with about 15-100mm coatings of wire mesh reinforced concrete, bitumen or fusion bond epoxy coatings(ASME B31.4, 2006) as illustrated in Figure 1. The process of intrusion involves access to the steel pipe by first disbondment of the coating followed by drilling through the pipe. Coating disbondment is achieved by hammering action; while drilling is a cutting operation.The mathematical model for predicting vandalism-induced failures has been formulated by idealizing a typical pipeline structure as a circular beam resting on an elastic Winkler-Pasternak soil foundation as shown in Figure 2.

2.1.2. Assumptions

In developing the model the following assumptions were made:

i. Damping is sufficiently small such that it can be ignored. Under steady state flow conditions hydrodynamic excitations, such as vortex-induced vibration due to hydrodynamic added mass, coriolis forces, internal surges etc. are also ignored.
ii. Soil loading, ground movements and other external loading of the pipeline are neglected. Phenomena such as punctures, outright buckling/collapse due to external loads and internal surges are excluded in the model.
iii. Non-conventional machining/cutting processes such as laser beam machining, electrical discharge machining, ultrasonic and chemical machining are excluded in the model.
iv. The model is only applicable to crude, condensate and petroleum product lines. The applicable materials are those allowable in ASME 31.4 code for high pressure cross-country pipelines (ASME, 2006).

2.1.3. Behaviour of Beams on Elastic Foundations

The analysis and characterization of dynamic response to intrusion forces on a typical pipeline structure involves the soil-pipeline structure on one hand and the pipeline structure-external load interaction on the other. Therefore, the proposed model takes account of the surrounding soil mechanical behaviour and resistance to loads. The pipe is idealized as resting on a Winkler-Pasternak foundation (Razaqpur *et al*., 1991; Cao *et al*., 2008). The stiffness and shear rigidity are the two parameters of the foundation as discussed in Razaqpur *et al*. (1991) thus

$$p(x)=k_{wf}w_1(x)-k_{pf}\frac{d^2w_1(x)}{dx^2} \tag{1}$$

Where the terms in equation (1) are as defined under notations

2.1.4. The Single Beam Model

The behaviour of the pipeline when invaded by vandals at any point along its length, can be idealized by taking some arbitrary point and cutting a differential transverse element of length *dx* at position *x*, as shown in Figures 3a, 3b and 4. The displacement $w_1(x, t)$ is measured from the static equilibrium position and is considered positive in the downward direction. Letting $V(x,t)$ and $M(x,t)$ be the shearing force and bending moment, considering the equilibrium forces and moments, the shearing force and bending moment acting on the two faces of the element are $V(x+dx,t) = V(x,t) + \partial V(x,t)dx/\partial x$ and $M(x+dx,t) = M(x,t) + \partial M(x,t)dx/\partial x$. The total transverse displacement of the beam is due to both bending and shearing deformations, so that the angle of rotation due to bending $\phi(x,t)$ and angle of distortion due to shearing $\varphi(x,t)$ of the beam (Figure 4) is related to the slope of the neutral axis by

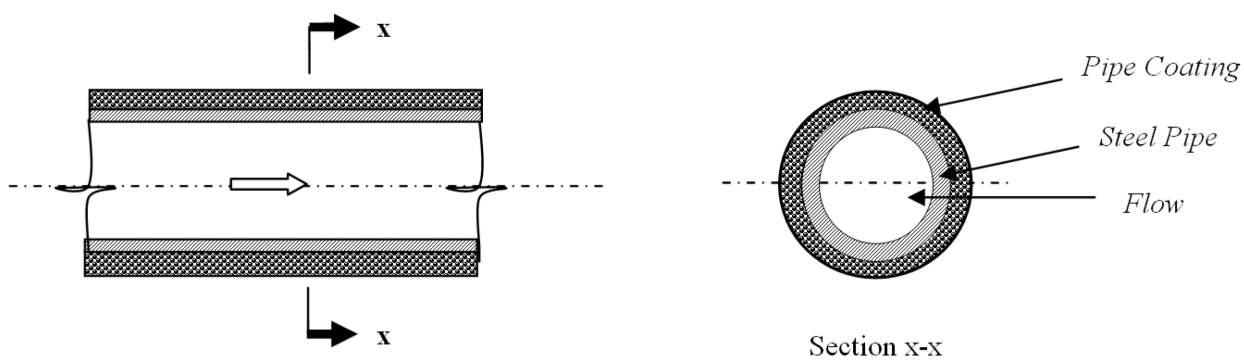

Figure 1: Typical Section of a Pipeline

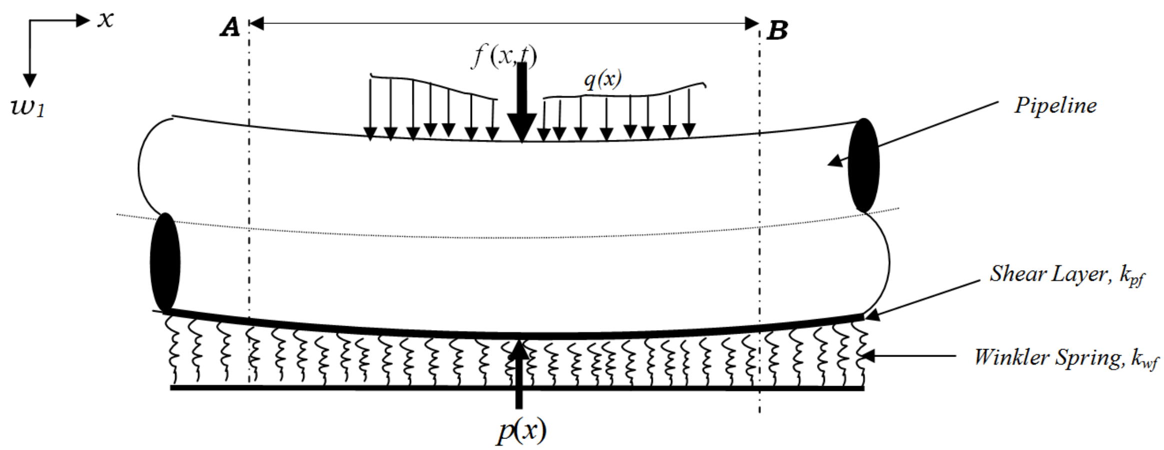

Figure 2: A Pipeline Idealized as a Beam Resting on Elastic Foundation

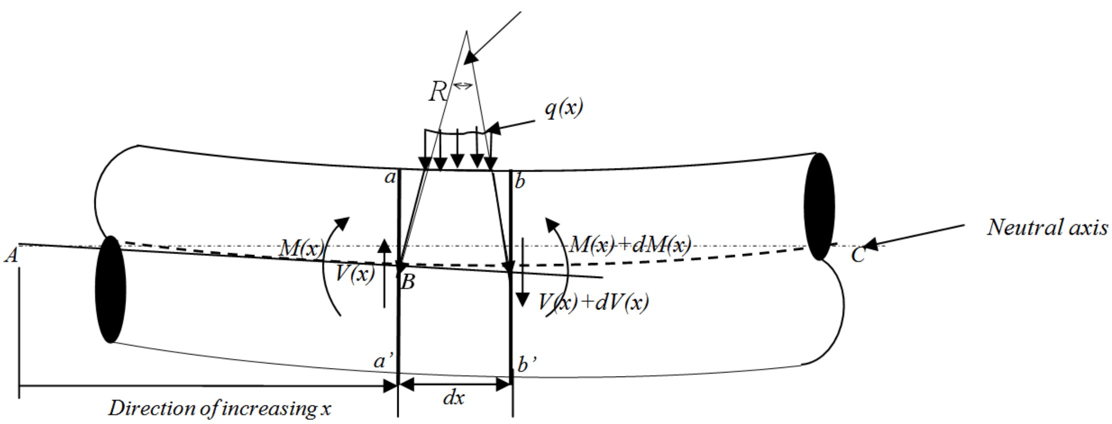

Figure 3a: Element of the Deflected Pipeline Beam Uncoupled from the foundation

$$\frac{\partial w_1(x,t)}{\partial x} = \varphi(x,t) + \phi(x,t) \quad (2)$$

$$\therefore\ \varphi(x,t) = \frac{\partial w_1(x,t)}{\partial x} - \phi(x,t) \quad (3)$$

De Rosa (1995) has shown that shearing force and the internal moment (assuming linear variations of strain and stress across the beam and ignoring damping effects) is related to shearing angle by

$$V(x,t) = kAG\varphi(x,t) = kAG\left(\frac{\partial w_1(x,t)}{\partial x} - \varphi(x,t)\right)$$

and

$$M(x,t) = EI_1\frac{\partial \varphi(x,t)}{\partial x} \quad (4)$$

k is the **Timoshenko shear coefficient**. In an earlier work, Foda (1998) has validated the factor experimentally to be k = 0.8333 for rectangular sections, 0.4167 for I- beams and 0.8475 for circular sections.

The spatial change in the bending moment and the shear force at $x + dx$ is

$$\frac{\partial M(x,t)}{\partial x} = -V(x,t) + \rho I_1 \frac{\partial^2 \varphi(x,t)}{\partial t^2} \quad (5)$$

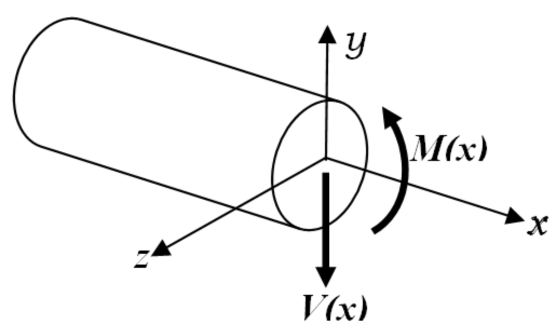

Figure 3b: 3-D Illustration of the moments and shear forces acting on the face aa' of element dx

From Newton's second law, $F = m_1 a = m_1 \frac{\partial v}{\partial t} = m_1 \frac{\partial^2 w_1(x,t)}{\partial t^2}$, the shearing force is related to the transverse acceleration by

$$\frac{\partial V(x,t)}{\partial x} = \rho A \frac{\partial^2 w_1(x,t)}{\partial t^2} - (q(x) + f(x,t)) \quad (6)$$

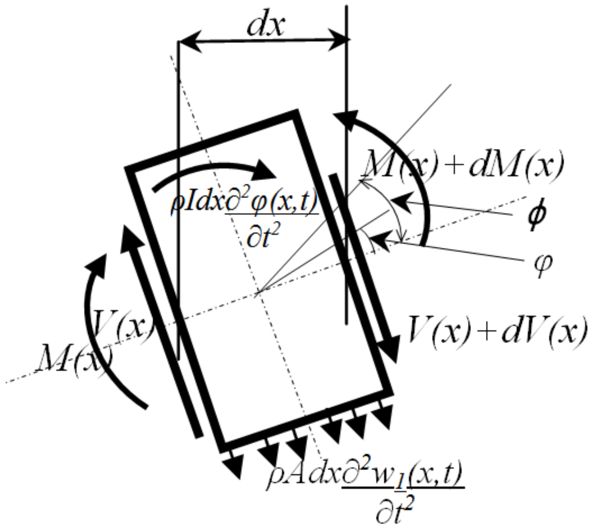

Figure 4: The moments and forces acting on an arbitrary element, dx

Differentiating $M(x,t)$ in equation (4) w.r.t. to x and equating to (5) yields:

$$EI_1\frac{\partial^2\varphi(x,t)}{\partial x^2}=-V(x,t)+\rho I_1\frac{\partial^2\varphi(x,t)}{\partial t^2} \quad (7)$$

Substituting $V(x,t)$ in equation (4) into (5) yields the **angular motion** thus:

$$EI_1\frac{\P^2 f(x,t)}{\P x^2}+kAG\left(\frac{\P w_1(x,t)}{\P x}-f(x,t)\right)-\rho I_1\frac{\P^2 f(x,t)}{\P t^2}=0 \quad (8)$$

Differentiating $V(x,t)$ in equation (4) w.r.t. to x yields:

$$\frac{\partial V(x,t)}{\partial x}=kAG\left(\frac{\partial^2 w_1(x,t)}{\partial x^2}-\frac{\partial\varphi(x,t)}{\partial x}\right) \quad (9)$$

Substituting equation (9) into (6) and rearranging yields the **transverse motion** thus

$$kAG\left(\frac{\partial^2 w_1(x,t)}{\partial x^2}-\frac{\partial\varphi(x,t)}{\partial x}\right)-\rho A\frac{\partial^2 w_1(x,t)}{\partial t^2}=-\left(q(x)+f(x,t)\right) \quad (10)$$

For the purpose of our problem, for equations (8) and (10) to be useful in analysis, it is desirable to couple the angular and the transverse motions into one equation of motion in terms of the displacement $w_1(x,t)$ by successive differentiation of equation (8) with respect to x and equation (10) with respect to t, and rearranging yields:

$$EI_1\frac{\partial^4 w_1(x,t)}{\partial x^4}+m_1\frac{\partial^2 w_1(x,t)}{\partial t^2}-\left(J+\frac{m_1}{kAG}EI_1\right)\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2}+\frac{m_1 J}{kAG}\frac{\partial^4 w_1(x,t)}{\partial t^4}=$$

$$\left(q(x)+f(x,t)\right)+\frac{J}{kAG}\frac{\partial^2 f(x,t)}{\partial t^2}-\frac{EI_1}{kAG}\frac{\partial^2}{\partial x^2}\left(q(x)+f(x,t)\right)$$

Where $\rho A=m_1$, the mass per unit length, $\rho I=J$, the polar moment of inertia. $q(x)$ denotes the load imposed by virtue of self weight due to line pipe material and fluid contents (could also include other permissible distributed loads on the pipeline such as backfill materials etc). $f(x,t)$ is the external vandalization point loads applied by vandals.

Since $k<1$, the quantities (EI_1/kAG) and (J/AG) are «1 and is thus ignored in this study. This is logical since the hammering and drilling forces applied by vandals on pipelines is not sufficiently large to cause significant turning moments, the Timoshenko beam theory thus converges towards:

$$EI_1\frac{\partial^4 w_1(x,t)}{\partial x^4}+m_1\frac{\partial^2 w_1(x,t)}{\partial t^2}-\frac{m_1}{kAG}\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2}=\left(q(x)+f(x,t)\right) \quad (12)$$

Adjoining the influence of the foundation, the deformation is equal and opposite to $\left[q(x)+f(x,t)\right]$

$$\therefore\quad p(x)=\left(q(x)+f(x,t)\right)=-\left(k_{wf}w_1(x)-k_{pf}\frac{\partial^2 w_1(x)}{\partial x^2}\right)+f(x,t) \quad (13)$$

Substituting equation (13) into the right hand side of (12) and rearranging yields

$$EI_1\frac{\partial^4 w_1(x,t)}{\partial x^4}+m_1\frac{\partial^2 w_1(x,t)}{\partial t^2}-\frac{m_1}{kAG}\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2}-k_{pf}\frac{\partial^2 w_1(x)}{\partial x^2}+k_{wf}w_1(x)=f(x,t) \quad (14)$$

The term $\left(-\frac{m_1}{kAG}\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2}\right)$ representing the influence of shearing deformation is negligibly small and ignored, so that equation (14) reduces to

$$EI_1\frac{\partial^4 w_1(x,t)}{\partial x^4}+m_1\frac{\partial^2 w_1(x,t)}{\partial t^2}-k_{pf}\frac{\partial^2 w_1(x)}{\partial x^2}+k_{wf}w_1(x,t)=f(x,t) \quad (15)$$

Equation(15) is the model relevant to our problem which describes the transverse response of the pipeline modelled as a beam resting on two-parameter Winkler-Pasternak foundation, under the influence of an external disturbing force, $f(x,t)$.

2.1.6. Solution of the Mathematical Model

The excitation force in equation (15) produces both free and forced vibrations of the system. The system can be solved either by **direct integration**, which is laborious, or by **numerical integration** to find the values of $w_1(x,t)$ over a solution field by using any of the well known schemes. In this paper, the classical solution of equation (15) was found and a step-by-step response approach (Weaver *et al.*, 1990) was used to compute the modal response at specified sampling times as now discussed.

For the **homogenous** part of equation (15), a solution is sought for

$$EI_1\frac{\partial^4 w_1(x,t)}{\partial x^4}+m_1\frac{\partial^2 w_1(x,t)}{\partial t^2}-k_{pf}\frac{\partial^2 w_1(x)}{\partial x^2}+k_{wf}w_1(x)=0 \quad (16)$$

which satisfies the equations in some region of the *x-t* plane in the form of a product of a function of *x* and of *t*, by separating the variables thus

$$w_1(x,t) = W_1(x)T(t) \quad (17)$$

Where $W_1(x)$ and $T(t)$ are space-dependent and time-dependent functions respectively. Thus, reducing the partial differential equation in two variables to an ordinary differential equation.

2.1.6.1. Free Vibration Response of the Pipe

Successive differentiation of equation (17), substitution into equation (16) and rearrangement, results in two equations viz:

(i) The **space-dependent** component of equation (16)

$$W_1''''(x) - \frac{k_{pf}}{EI_1} W_1''(x) - \left(\frac{\omega_a^2 m_1 - k_{wf}}{EI_1}\right) W_1(x) = 0 \quad (18)$$

From an assumed solution of $W_1(x) = Ae^{px}$, the auxiliary equation $p^4 - \frac{k_{pf}}{EI_1} p^2 - \left(\frac{\omega_a^2 m_1 - k_{wf}}{EI_1}\right) = 0$ is obtained which yields four roots, hence the general solution of eq (18) becomes

$$W_1(x) = A_1 e^{s_1 x} + A_2 e^{-s_1 x} + A_3 e^{js_2 x} + A_4 e^{-js_2 x} \quad (19)$$

Introducing trigonometric and hyperbola identities, equation (19) can be expressed in its equivalent form (Rao, 2007 and Weaver *et al*., 1990) as

$$W_1(x) = A_1 \cosh s_1 x + A_2 \sinh s_1 x + A_3 \cos s_2 x + A_4 \sin s_2 x \quad (20)$$

Equation (20) satisfies equation (18) and it is therefore a valid solution field.

(ii) The **time-dependent** component of equation (16)

$$\ddot{T}(t) + \omega_a^2 T(t) = 0 \quad (21)$$

and solution is of the form:

$$T(t) = B\cos\omega_a t + C\sin\omega_a t \quad (22)$$

Now substituting equations (20) and (22) into $w_1(x,t) = W_1(x)T(t)$ yields the complete solution

$$w_1(x,t) = \left(A_1 \cosh s_1 x + A_2 \sinh s_1 x + A_3 \cos s_2 x + A_4 \sin s_2 x\right)\left(B\cos\omega_a t + C\sin\omega_a t\right) \quad (23)$$

Where $s_1 = p_1 = -p_2$ and $js_2 = p_3 = -p_4$;

$$p^2 = \frac{k_{pf}}{2EI_1} \pm \sqrt{\frac{k_{pf}^2}{4E^2 I_1^2} + \left(\frac{\omega_a^2 m_1 - k_{wf}}{EI_1}\right)}$$

A_1, A_2, A_3, A_4, are constants to be determined from prescribed boundary conditions, while B and C are to be determined from initial conditions. The function $W_1(x)$ is the shape of the natural modes/**principal function.** *s* includes both the flexural rigidity of the structure and the elasticity of the supporting medium.

2.1.6.2. Natural Frequencies and Mode Shapes of Pipe

The pipe has been idealized as a continuous beam assumed to be simply supported at multiple points on an elastic foundation. Essentially for a typical span, the conditions connoting the deflection and bending moment of the beam at each pair of boundary are as follows:

(i) $(W_1)_{x=0} = 0$; (ii) $EI_1\left(\frac{d^2 W_1}{dx^2}\right)_{x=0} = 0$; (iii) $(W_1)_{x=l} = 0$ and (iv) $EI_1\left(\frac{d^2 W_1}{dx^2}\right)_{x=l} = 0$

Successive differentiation and substitution of the boundary conditions into equation(20) yields the system

$$A_2 \sinh s_1 l + A_4 \sin s_2 l = 0$$

$$A_2 s_1^2 \sinh s_1 l - A_4 s_2^2 \sin s_2 l = 0 \quad (24)$$

For non-trivial solutions of A_2 and A_4, the 2 x 2 determinant of their coefficient matrix must vanish thus

$$\begin{vmatrix} \sinh s_1 l & \sin s_2 l \\ s_1^2 \sinh s_1 l & -s_2^2 \sin s_2 l \end{vmatrix} = 0$$

Expansion of the determinant produces the **frequency equation** $\sinh s_1 l \sin s_2 l = 0$, a condition which is satisfied when $s_{2n} l = n\pi$; Þ $s_{2n} = (n\pi/l)$. The mode shape corresponding to the angular frequency ω_a is given by

$$W_1(x) = A_n \sin(n\pi/l)x \quad (25)$$

A_n is a new constant being the ratio of A_4/A_2 in equation (24), which is assumed to be unity throughout the remainder of this paper for simplicity and computational efficiency. By successive differentiation of equation (25) and substitution into (18) we obtain the corresponding circular frequency of the nth mode as

$$\omega_{an} = \left(\frac{n\pi}{l}\right)^2 \sqrt{\frac{EI_1}{m_1}} + \sqrt{\frac{(n\pi)^2 k_{pf} + l^2 k_{wf}}{m_1 l^2}} \quad [\text{rad/sec}]$$

n=1,2,3... (26)

Thus, from equation (23), the total dynamic response of the beam will be the superposition of all the normal modes as follows

$$w_1(x,t)=\sum_{n=0}^{\infty}W_{1n}(x)(B_n cos\omega_{an}t+C_n sin\omega_{an}t) \quad (27)$$

Where $W_{1n}(x)=sin(n\pi/l)x\ ;\ n=1,2,3,...$; n = the mode number.

2.1.6.3. Forced Vibration Response

The response to the time-varying fluctuating load imposed externally on the pipe is now investigated. In order to obtain the total response of equation (15), the solution is considered using the normal mode approach (Rao, 2007), whereby the transverse displacement field is assumed to be a linear combination of the normal modes of the structure in the form of a finite series as shown in equation(27) thus

$$w_1(x,t)=\sum_{n=1}^{\infty}W_{1n}(x)\phi_n(t) \quad (28)$$

In which $W_{1n}(x)$ is the normal function evaluated at different points in x which satisfies equation(18), while $\phi_n(t)$ is modal participation coefficients in terms of time function or the *nth* modal coordinate. Equation(28) implies that at any arbitrary given time(t>0), the function $w_1(x,t)$ can be approximated by linear combination of $W_{1n}(x)$ and ϕ_n(t). The process leads to a set of second order equations in terms of the generalized coordinates, which can be solved by using the initial conditions of the problem as now discussed. Successive differentiation of equation (28) and substitution into (15) yields

$$\sum_{n=1}^{\infty}\left[\omega_a^2 W_{1n}(x)\right]\phi_n(t)+\sum_{n=1}^{\infty}W_{1n}(x)\frac{d^2\phi_n(t)}{dx^2}=\frac{f}{m_1}(x,t)=q(x,t) \quad (29)$$

Applying the concept of orthogonality, substitutions and rearrangements we obtain

$$\frac{d^2\phi_n(t)}{dt^2}+\omega_a^2\phi_n(t)=q_n(t);\quad n=1,2,3,...\ ,$$

$$q_n(t)=\int_0^l W_{1n}(x)q_n(x,t)dx \quad (30)$$

The differential equation (15) has thus been transformed to normal coordinates by substituting equation (28) for $w_1(x,t)$. The complete solution of equation (30) is

$$\phi_n(t)=B_n\cos\omega_{an}t+C_n\sin\omega_{an}t+\frac{1}{\omega_{an}}\int_0^t q_n(t')\sin\omega_{an}(t-t')dt' \quad (31)$$

Hence the total response of the beam to external disturbance is

$$w_{1n}(x,t)=\sum_{n=1}^{¥}W_{1n}(x)\left\{B_n\cos\omega_{an}t+C_n\sin\omega_{an}t+\frac{1}{\omega_{an}}\int_0^t q_n(t')\sin\omega_{an}(t-t')dt'\right\} \quad (32)$$

Where

$W_{1n}(x)(B_n\cos\omega_{an}t+C_n\sin\omega_{an}t)$ denotes the homogeneous solution or free response as in equation (27). B_n and C_n are constants to be determined from initial conditions.

$W_{1n}(x)\left(\frac{1}{\omega_{an}}\int_0^t q_n(t')sin\omega_{an}(t-t')dt'\right)$ is the forced response.

$W_{1n}(x)$ is the modal shape of the response, $q_n(t)$ is the nth normal-mode load and the integral term over $q_n(t)$ in which we assume that the force , $q_n(t)$ is expressed as a function of a dummy time t' variable (Duhamel's integral).

2.1.7. Control problem formulation

Having modelled the pipeline structure, an appropriate feedback control algorithm was formulated such that the response generated from the induced loading can be detected in a controlled and quantifiable manner. The motion described by equation (15) is required to be satisfied over the domain, as well as end conditions to be satisfied at the boundaries of each domain. Therefore, the control problem can be characterized as that of distributed parameter systems. By using the expansion theorem to obtain equation (32), we essentially replaced the respective partial differential equations governing the motion of the pipe by an infinite set of ordinary differential equations (30). The feasibility of a feedback control algorithm for the pipe is now discussed.

2.1.7.1. Closed-Loop Modal State Equations of the Pipe

Rewriting the configuration equation (30) in **matrix form** as

$$\ddot{\varphi}_n(t)+\Omega_{an}^2\varphi_n(t)=B_{an}q(t) \quad (33a)$$

Assuming the system to be linear, the output law of the system can be defined as

$$y_a(t)=\mathcal{C}_{an}\varphi_n(t) \quad (33b)$$

Where

$\varphi_n(t)=\left[\varphi_1(t)\,\varphi_2(t)\ \ ...\varphi_n(t)\right]^T$ = nth modal or normal generalized coordinates/configuration vector

$\Omega_{an}^2=diag\left[\omega_{an}^2\right]$ =infinite-order diagonal matrix of natural frequencies

$B_{an}=\left[b_{a1}\ \ b_{a2}\ \ ...\ \ b_{an}\right]^T$ = Input (control) matrix

$\mathcal{C}_{an}=\left[c_{a1}\ \ c_{a2}\ ...\ c_{an}\right]$ = output matrix

$q(t)$ =input signal (applied disturbance force)

$y_a(t)$ = output signal

The solution $w_{1n}(x,t)=\sum_{n=1}^{\infty}W_{1n}(x)\varphi_n(t)$ which gave rise to equation (32) implies the entire infinity of modes need to be controlled which is not feasible. Therefore, it must be truncated at some *rth* mode to yield

$$w_{1n}(x,t)\cong\sum_{n=1}^{r}W_{1n}(x)\phi_n(t) \quad (34)$$

Substituting equation (34) into equation (30) yields

$$\ddot{\varphi}_n(t)+\omega_{an}^2\varphi_n(t)=q_n(t) \qquad (35)$$

Where $n = 1, 2, ..., r$ and $q_n(t)=\int_0^l W_{1n}(x)q_n(x,t)dx$

Equation (35) represents a set of r -simultaneous second-order ordinary differential equations in which the infinitely-many-degree-of-freedom system has now been approximated by an r-degree-of-freedom system by truncation. Such a control scheme as equation (35) in which every mode is individually controlled; is the so-called independent modal space control (IMSC) method. Similar schemes have been applied for parameter distributed systems problems in other fields, especially for maintaining asymptotic stability of such systems like large scale structures, robots, satellites etc. (Meirovitch, 1990; Balas, 1978).

The modal response in equations (35) are not physical coordinates and cannot be measured directly, hence we need to devise a means of extracting the modal coordinates from measured data. Now if we introduce a device permitting sensing of the physical displacements, velocities and/or accelerations (say by discrete sensors), the modal response can then be obtained, albeit approximately. Letting $y_{ai}(t)$ be the outputs from such sensors and assuming the displacements of the structure can be obtained at points x_i, $i = 1, 2, 3, ..., s$, the modal coordinates can be estimated, in which the sensors measures only the contribution of the controlled modes to the motion of the structure. The output vector if displacement feedback is used is

$$y_{ai}(t)=w_1(x_i,t)=\sum_{n=1}^{r}W_{1n}(x_i)\phi_n(t) \qquad (36a)$$

If velocity sensors are used we have velocity feedback as

$$\hat{y}_{ai}(t)=\dot{w}_1(x_i,t)=\sum_{n=1}^{r}W_{1n}(x_i)\dot{\phi}_n(t) \qquad (36b)$$

And if acceleration sensors (accelerometers) are employed acceleration feedback as

$$\breve{y}_{ai}(t)=\ddot{w}_1(x_i,t)=\sum_{n=1}^{r}W_{1n}(x_i)\ddot{\phi}_n(t) \qquad (36c)$$

Where the approximate measurands, $y_{ai}(t), \hat{y}_{ai}(t)$ and $\breve{y}_{ai}(t)$; represent the displacement, velocity and acceleration, respectively.

3. MODEL APPLICATION

For our problem (i.e. pipeline vandalization), we can now specify the external disturbing force *f(x,t)* in equation(15). Impact blows from sledge or jack hammers during coating disbondment, as well as thrust force and cutting torques from rotary cutting tools were identified as the important input loads in the intrusion process. In the case intrusive cutting force the response can be determined as now discussed.

3.1. Cutting Forces

Cutting thrust force and torque from rotary tools are the output loads during the hot tapping involved in the vandalization process. In the case of drilling cutting force, for instance, let the disturbing force induced by intruders during drilling of the pipe be equal to the cutting force F_c varying harmonically at angular frequency Ω_v. F_c is applied as a point load at some point, $\hat{a}$ from the left support. The force acting on a typical segment is treated as a spatial Dirac delta function as shown in Figure 5. $f(x,t)$ will thus take the form:

$$f(x,t)=F_c\sin\Omega_v t.\delta(x-\hat{a})$$

$$\therefore\ q(x,t)=\frac{F_c}{m_1}\sin\Omega_v t.\delta(x-\hat{a})=q_c\sin\Omega_v t.\delta(x-\hat{a}) \qquad (37)$$

Where F_c = harmonically varying cutting force applied at $x = \hat{a}$.

As in preceding discussions, the generalized force corresponding to the nth mode is

$$q_n(t)=\int_0^l W_{1n}(x)q(x,t).\delta(x-\hat{a})\,dx$$

$$n \quad q_n(t)=q_c W_{1n}(\hat{a})\sin\Omega_v t \qquad (38)$$

Substituting equation (38) into (31) yields

$$\varphi_n(t)=B_n\cos\omega_{an}t+C_n\sin\omega_{an}t+\frac{q_c}{\omega_{an}}\int_0^t W_{1n}(\hat{a})\sin\Omega_v t\sin\omega_{an}(t-t')dt'$$

Noting the trigonometric identities

$$\sin\Omega_v t\sin w_{an}(t-t')=\frac{1}{2}\left\{\begin{array}{l}(\cos(\Omega_v t-w_{an}(t-t'))\\ -\cos((\Omega_v t+w_{an}(t-t'))\end{array}\right\}$$

and integrating we have

$$\varphi_n(t)=B_n\cos\omega_{an}t+C_n\sin\omega_{an}t+\frac{q_c}{\omega_{an}^2}W_{1n}(\hat{a})\left(\sin\Omega_v t-\frac{\Omega_v}{\omega_{an}}\sin\omega_{an}t\right)\frac{1}{1-\Omega_v^2/\omega_{an}^2} \qquad (39)$$

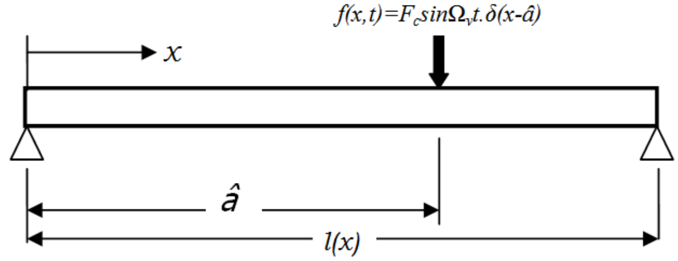

Figure 5: Illustration of Cutting Tool acting on a Pipeline Segment

The total response of the pipe is found by substituting equation (39) into equation (32) to obtain

$$w_{1n}(x,t)=\sum_{n=1}^{\infty}W_{1n}(x)\left\{\begin{array}{l}B_n\cos\omega_{an}t+C_n\sin\omega_{an}t+\frac{q_c}{\omega_{an}^2}W_{1n}(\hat{a})\\ \left(\sin\Omega_v t-\frac{\Omega_v}{\omega_{an}}\sin\omega_{an}t\right)\frac{1}{1-\Omega_v^2/\omega_{an}^2}\end{array}\right\}$$

(40)

Practically, the first part of equation (40) represents the natural vibration, the second part denotes the steady-state forced vibrations induced on the pipeline, while the third part consists of transient free vibration, which peters out with time. Therefore, only the steady-state response is of practical implication in our problem.

Now, if we prescribe the initial conditions $\varphi_n(0)=0; \dot{\varphi}_n(0)=0$, B_n and C_n are determined. The term $\left(\frac{1}{1-\Omega_v^2/\omega_{an}^2}\right)$ in equation (40) denotes the magnification factor (MF). Since cutting force is varying slowly, Ω_v is small relative to ω_{an}, therefore $MF \approx 1$ and equation (39) reduces to

$$\phi_n(t) = \frac{q_c}{\omega_{an}^2 - \Omega_v^2} W_{1n}(\hat{a}) \left(sin\Omega_v t - \frac{\Omega_v}{\omega_{an}} sin\omega_{an} t \right) \quad (41)$$

From equation (41) and truncating equation (40) at the rth mode yields

$$w_{1n}(x,t) = q_c \sum_{n=1}^{r} W_{1n}(x) \left(\frac{W_{1n}(\hat{a})}{\Omega_v^2 - \omega_{an}^2} \left(sin\Omega_v t - \frac{\Omega_v}{\omega_{an}} sin\omega_{an} t \right) \right) \quad (42)$$

Where $\sin(n\pi x/l) = W_{1n}(x)$ and $\sin(n\pi\hat{a}/l) = W_{1n}(\hat{a})$.

3.2. Application Example

Considering a pipeline segment in Figure 5, in attempt to perforate the pipe during a typical intrusive attack, at some point x lying in $[0 \le 1.28 \le 2]$ m, a twist drill mounted on a magnetic drilling machine is used to initiate a 6 mm diameter pilot hole to a depth approximately 60% of the pipe wall thickness. Prior to final perforation, using 10-25 mm diameter hole cutters, the pilot hole is successively enlarged until the internal diameter of the 25 mm branch fitting installed is achieved. The maximum thrust feed force impacted on the pipeline during the intrusion is 45 kN at a cutting speed of 78.54 rad/sec (750 rpm). The pipeline structure is assumed to be initially at rest and the relevant boundary conditions are those specified in paragraph 2.1.6.2. Furthermore, the pipeline is assumed to be laid on a soil foundation that has Winkler and Pasternak moduli of 1062 kN/m2 and 14920 kN respectively. It is desired to use a single point sensor located at 1.15 m from the left support to estimate the behaviour associated with the three lowest controlled modes. The pipeline parameters (similar to the specimens investigated in the main research) are as summarized in Table 1.

Table 1: Specification of Pipe Segment

Parameter	Data
Pipeline Material	API 5L Gr. X52. [0.31% carbon steel seamless pipe with specified minimum yield strength of 354.50 x 103 kPa and Charpy impact energy above 292 J at room temperature].
Service Fluid	Crude Oil
Nominal size of pipe	250 mm
Outside Diameter, Do	273.05 mm
Internal Diameter, Di	254.51 mm
Pipe wall thickness	15.09 mm (Sch. 80)
Young Modulus of pipe material	210 x106 kPa
Density of Pipe Material	7850 kg/m^3
Density of Fluid	848 kg/m^3 (Nigerian Light)
Winkler Soil Modulus, kwf	1062 kN/m^2 or kPa/m^2
Patersnak Modulus, kpf	14920 kN

4. NUMERICAL SIMULATION AND DISCUSSION OF RESULTS

For the problem stated in paragraph 3.1.2, the modal model (33a) is constructed directly from equation (15). Using separation of variables equation (28), which is truncated at the rth term (i.e. r = 3), the modal coordinates are computed using a step-by-step response approach(Weaver et al., 1990) from equation (41) at sampling times (0.00, 0.20,...,2.0) s. The input(control) and output influence matrices in equation (33) are obtained respectively from Bar = W1r(a) = sin(rπ1.28/l) and Ȼar = sin(rπ/l)(xi) where r = 1,2,3 and i = 1,2,...,5. Substituting the initial conditions $\phi(0)=0$ and $\dot{\phi}(0) = 0$ into $\varphi_n(t)$ yields equation (41). At sampling times (0.00, 0.20, 0.40, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0) sec. the modal displacement feedback is obtained as presented in Figure 6, while the combined modal response is as in Figure 7.

As seen in Figures 6 and 7, consistent with the assumption in truncating the modal model, high frequency components (n>2) do not contribute significantly to the response. The maximum response for each mode is obtained at around 1.2 sec. This is the steady state of the system and in line with the comments highlighted in paragraph 3.0.

Using the linear feedback control law equation (33b) and sensor functions in equation(36a), the output displacement vector for each mode at the sampling times are obtained. Each pair of the W1n(xi)ϕr(tj) corresponds to the response at sample points randomly selected for the simulation at (0.00, 0.60, 1.10, 1.40 and 2.00) m as presented in Figure 8. The displacement response indicates maximum response at around the point of attack and zero deflection at the both end which is consistent with the boundary conditions.

The theoretical formulation of this work has been verified in the application example. Further study involving large scale experimental validation of the model on a prototype using various forms of sensors is currently under investigation.

5. CONCLUSION

This work indicates that the method of discrete sensors to extract modal coordinates/velocities/ accelerations is promising and a potential approach to solving the problem of intrusion on pipeline under the model and control algorithm presented in this paper. Existing models such as those for leak detection are post event procedures which acknowledge that the damage has already occurred. Therefore, a truly and robust predictive model relevant to intrusive attack should be such that accounts for pre-event scenarios as presented in this paper. This can function in tandem with conventional models such as those for leak detection, corrosion etc to enhance overall pipeline safety.

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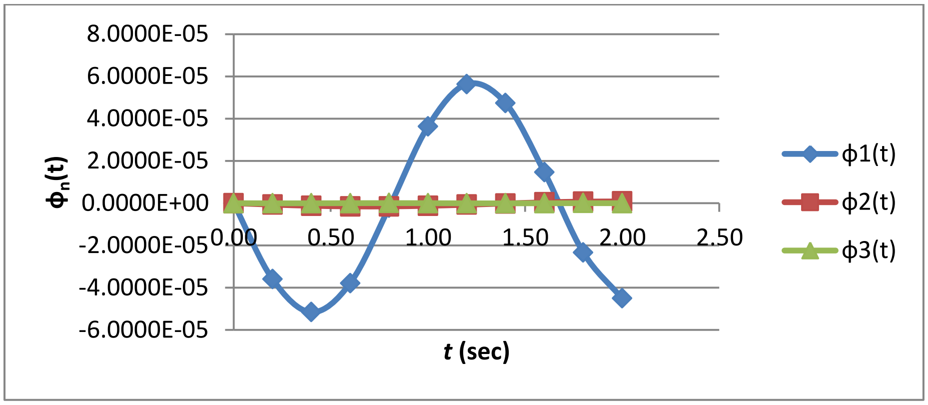

Figure 6: Modal Response of the Controlled Mode to Intrusive Cutting Force

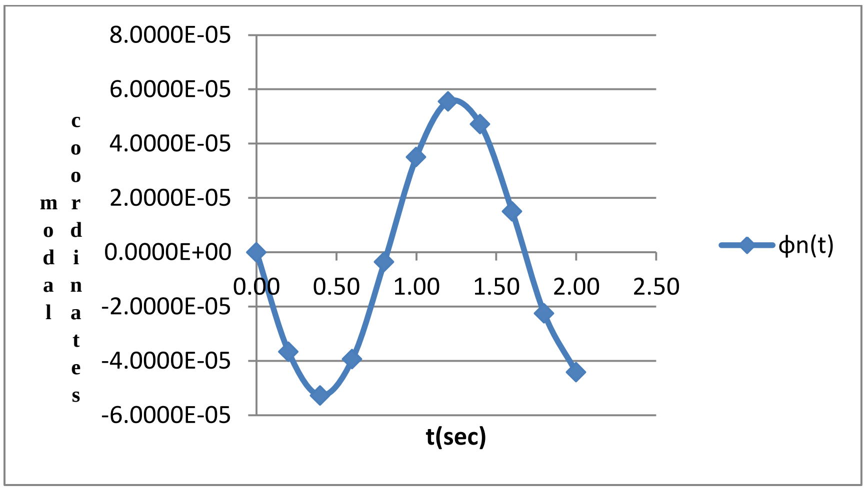

Figure 7: Combined Dynamic Response of the Controlled Mode to Intrusive Cutting Force

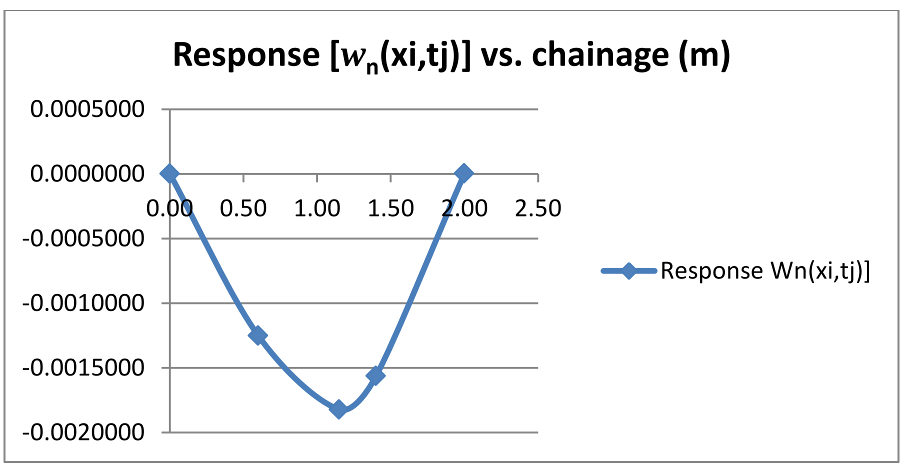

Figure 8: Dynamic Response vs. length to Intrusive Cutting Force

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NOTATIONS

A	cross sectional area of the pipe (m^2)
E	Young's Modulus of the constitutive pipe material (Pa)
G	Modulus of rigidity or Shear modulus
I	second moment of area (area moment of inertia) of the pipe cross- section the about the neutral axis (m^4).
J	polar second moment of area (m^4)
l	length (m)
M	bending moment
V	shear force
R,r	radius
t	thickness, any particular instant of time
f(x,t)	external excitation force [e.g. vandal induced forces]
φ(x, t)	rotation of cross-section about neutral axis; θ (x,t) Angle of twist.
ρ	mass density of the pipe and liquid coupled together(kg/m^3)
q(x)	Uniformly distributed transverse force acting per unit length due to mass of the pipe(mp) and mass per unit length of the fluid(mf). m1 sum of the masses of pipe and fluid (mp+mf) per unit length
x	the co-ordinate along the neutral axis
$w_1(x, t)$	transverse displacement (deflection) function of the beam from an equilibrium state w.r.t. the neutral axis.
kwf	first foundation parameter(Winkler foundation modulus)[N/m2]
kpf	second foundation parameter of the soil shear layer (Pasternak foundation modulus) [N]
p(x)	the vertical foundation reaction along x